Dark Matter as a Coherent Wave
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Motivations:

- Missing satellite problem
  
  CDM predicts there should have been orders of magnitude more dwarf galaxies of \(10^6 - 10^8\) M\(_\odot\) as satellite galaxies of Milky Way
  
  **Not observed!**

- Flat core problem
  
  In those rare dwarf spheroid satellite galaxies, the core profiles do not agree with the NFW profile
Motivations:

- Too big to fail problem

Large dwarfs have a sufficiently deep gravitational potential to keep the baryons, according to NFW.

But baryons are missing in the potential

- Void problem

According to CDM, voids depend on galaxy masses, and big voids should be traced by massive galaxies.

But in the 15Mpc local void it is observed devoid of dwarf galaxies

- All puzzles pertain to dwarf galaxies ……
**φ DM (or fuzzy dark matter)**

- $m_a = 10^{-22}$ eV, $n_0 = 10^{25}$/cc, $T_c >$ MeV

- Bose-Einstein condensed state, described by a single wave function $\psi$. When written in the co-moving coordinate

$$\left[ i \frac{\partial}{\partial \tau} + \frac{\nabla^2}{2} - aV \right] \psi = 0$$

where $d\tau = dt/a^2$

- $\nabla^2 V = 4\pi(|\psi|^2 - 1)$

- Self-similar scaling relations and Jeans length

$\tau \rightarrow \lambda^2 \tau$, $r \rightarrow \lambda r$, $n \rightarrow \lambda^{-4} n$, $M \rightarrow \lambda^{-1} M$, equations do not change
Linear power spectrum at $z=0$

$P(k) \left[ \left( h^{-1} \text{Mpc} \right)^3 \right]$ vs $k \left[ h \text{ Mpc}^{-1} \right]$

- CDM
- $m_a = 10^{-22} \text{ eV}$

Jeans length
$\sim a^{-1/4}$

AMR calculations:

- This equation is very easy to solve using FFT in fixed grid

- But when using AMR, this equation has unfavorable scaling, $\Delta t \sim \Delta x^2$, reflecting $\omega \sim h^2 k^2 / 2m$ for matter waves

- Use GPU to accelerate the number crunching and use CPU to control the logic flows
AMR calculations:

Dark matter as a coherent quantum wave
AMR calculations:
Core-Halo relation

$z=0$ dwarf galaxies

Redshift evolution
Core-Halo relation

- Our primary goal is to establish $r_{\text{core}}$, $M_{\text{core}}$, and $M_{\text{halo}}$ relation

- This allows us to predict the hard-to-observed core if we have an estimate of the halo mass.

- For example, $r_{\text{core}} \sim 150$ pc and $M_{\text{core}} \sim 10^9 \, M_\odot$ for Milky Way size galaxy of $M_{\text{halo}} \sim 10^{12} \, M_\odot$
Core-Halo relation: $M_c$ depends on conserved mass and energy?
Core-Halo relation: \[ M_c \sim M_{\text{halo}}^{1/3} a^{-1/2} \]

\[ a^{1/2} M_c (M_\odot) \]

\[ (\zeta(z)/\zeta(0))^{1/2} M_h (M_\odot) \]

10 > z > 2.5
2.5 > z > 1.5
1.5 > z > 0.6
0.6 > z > 0.2
0.2 > z > 0.0

Single halo (3.3 > z > 0)

arXiv: 1407-7762
SMBH-Core-Halo connection

- Tight $M_{bh} - \sigma_h^4 \sim M_{halo}^{4/3}$ relation requires a midsize agent, since the two length scales differ by $10^{10}$
SMBH-Core-Halo connection

- Solitonic core (100 pc, $10^9 M_\odot$) lies in the midway between virial radius (300kpc) and Schwartzchild radius (0.0001pc), providing an intermediate size funnel for gas accumulation and accretion
- $M_{\text{halo}}$ has a good correlation with $M_c$, and the question is boiled down to what $M_{\text{bh}} - M_c$ may be
- The core funnel force, $GM_c/r_c^2 \propto (1+z)^{3/2}$. For a $z=10$ galaxy, the funnel force is 35 times larger than that of a $z=0$ galaxy, thus helps triggering the prompt formation of quasars at $z=7$
Strong lens quad image flux anomaly

\[ R = \frac{\mu_A + \mu_B + \mu_C}{|\mu_A| + |\mu_B| + |\mu_C|} \]

CDM

\[ \mu_D = -3.925 \]
\[ \mu_E = 0.291 \]
\[ \mu_A = 17.840 \]
\[ \mu_B = -30.281 \]
\[ R = 0.08 \]

\[ \mu_C = 17.680 \]

\( \psi \)DM

\[ \mu_D = -4.317 \]
\[ \mu_E = 0.133 \]
\[ \mu_A = 39.992 \]
\[ \mu_B = -22.371 \]
\[ R = 0.37 \]
\[ \mu_C = 8.629 \]
Why is $\psi$DM boson mass $10^{-22}$ eV?

Tilted wine bottle (axion) model + one general assumption about the dark sector naturally yields

$$m_a \sim 10^{-22} \text{ eV}$$
$$f_a \sim 4 \times 10^{16} \text{ GeV}$$
$$m_\Lambda \sim 100 \text{ eV}$$

*arXiv:1409-0380*
Why is $\psi$DM boson mass $10^{-22}$ eV?

- $T = 200$ GeV
  - All particles in the same soup

- Relativistic $\Lambda$ dark particles + dark photon

- Bright sector

- Particle annihilation when non-relativistic
Why is $\psi$DM boson mass $10^{-22}$ eV?

0.2 MeV > $m_\Lambda > 0.3$ eV

Td ~ (1/3) Tb ~ $a^{-4}$

- Non-relativistic $\Lambda$ dark particles annihilate to create axion oscillation
- Non-relativistic axion density ~ $a^{-3}$
- 3 species of neutrinos + Photons

Present Universe $\Omega_m = 0.26$

Matter-Radiation Equality
Conclusion:

- The cores of $\psi$DM can begin to explain the existing dwarf galaxy problems
- The compact core of $\psi$DM in massive galaxies have a good potential to link to formation of SMBH
- The granules of $\psi$DM can create the flux anomaly of strong lensing quad images
- $10^{-22}$ eV is an interesting mass scale, which naturally arises from dark-sector axion model