Formation of Nuclear Ring from Hydrodynamic Simulations using Multipole Expansions

Preliminary result

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To simulate the sub-galactic scale structure formation in the Lambda CDM model, we have developed a new cosmological hydrodynamic code. (Shin, Kim, Kim, & Park 2014)

\[ \text{E} \text{U} \text{N} \text{HA} : \text{Evolution of the U} \text{niverse simulated with } N \text{-body and Hydrodynamic Algorithms} \]

- based on the most efficient code (PM+tree) for the large scale structure, \textit{GOTPM} (Dubinsky, Kim, & Park 2003)
- improved the hydrodynamics (SPH) into the GOTPM code (mainly by Juhan Kim)
- used a time-step limiter scheme (Saitoh & Makino 2009) over the individual time step
- added the realistic baryonic physics
  - Reionization process by UV sources and UV shielding (Haardt & Madau 1996, Sawala et al. 2010)
  - Radiative heating/cooling (T ~ reach to 100K) using CLOUDY 90 package (Ferland et al. 1998)
  - Star formation as single stellar population (Katz 1992, Abadi et al. 2003, Kroupa 2001)
  - Metal and energy feedback by \textit{SN} \textit{II} (Hurley, Pols, Tout 2000, Okamoto, Nemmen, & Bower 2008, Woosley & Weaver 1995)
To simulate the sub-galactic structure formation/evolution in non-cosmological frame, we put the astrophysical gas processes (Shin, Kim, Kim, & Park 2014) into GADGET-2 (Springel 2005).

- radiative heating/cooling, reionization, shielding, star formation, and SN feedback
- mainly by Kyungwon Chun (see poster, P-1)

Research plans

- formation/evolution of CMZ (central molecular zone), nuclear bulge, and CND (circumnuclear disk)
- formation of sub-galactic structures (globular clusters, ultra compact dwarf galaxies) in cosmological frame

Code development plans

- based on Gadget-3 for better scalability
- including multiphase model
- including different star formation efficiency for different gas density (Kruijssen 2012)
Nuclear Star-Forming Rings in External Galaxies

Mazzuca et al. 2008

Morris & Serabyn 1996
The Milky Way also has a Bar

Circumnuclear Disk

Mann & Thaddeus, 2001)
Migration of Gas toward the Galactic Center

1. Non-axisymmetric potential (nested bar and/or nuclear bulge)
2. Fluctuating motions in the CMZ
3. Magnetic viscosity
4. Dynamical friction
Recent Numerical Simulations


-> 3D hydrodynamic simulation using particle-based code (ASURA, Saitoh et al. 2008, 2009)
  + including radiative cooling, star formation, SN feedback
  + galactic potential: m=2 bar with a power-law density profile for the inner Galactic bulge

Kim, W.-T. & Seo, W.-Y., & Kim, Y., 2012

  + galactic potential: non-axysymmetric stellar bar (Ferrers 1987), stellar disk, stellar bulge, central BH
  + formation and evolution of gaseous substructures in barred galaxies with varying bar strength

Seo, W.-Y., & Kim, Y.-T., 2013

-> Based on Kim, W.-T. & Seo, W.-Y., & Kim, Y., 2012
  + including star formation and SN feedback

Seo, W.-Y., & Kim, Y.-Y., in press

-> based on Seo, W.-Y., & Kim, Y.-T., 2013
  + galactic potential: spiral arms
Our Strategy on Numerical Simulations

3-dimensional hydrodynamic simulation

+ realistic baryonic physics (including radiative cooling/heating, star formation, SN feedback)
+ realistic mass inflow toward the Galactic center from the disk
+ long term evolution to see formation of the nuclear bulge
+ realistic galactic potential of the Galaxy

How do we consider the realistic galactic potential of the Milky Way?
Simulation snapshots for the Milky Way-like galaxy model (of J. Baba & T. Saitoh)

( ~10^7 gas and star particles for both disk and bulge components + fixed halo model)

+ multipole expansions for modeling the realistic galactic potentials
Previous Numerical Studies with Multipole Expansions

1. 2-dimensional, cylindrical model (Aoki & Iye 1978) : disk only system

\[ \mu_{nm}(R, \phi) = \frac{Ma}{R_a^3} \frac{(2n+1)}{2\pi} P_{nm}(\xi) \exp(i m \phi) \]

\[ \Phi_{nm}(R, \phi) = -\frac{GM}{R_a} P_{nm}(\xi) \exp(i m \phi) \]

2. 3-dimensional, cylindrical model (Hozumi & Hernquist 2005) : numerical problems

\[ \mu_{nm}(R) = \frac{2n + 1}{2\pi} \left( \frac{1 - \xi}{2} \right)^{3/2} P_{nm}(\xi) \exp(i m \theta) \]

\[ \Phi_{nm}(R) = -\left( \frac{1 - \xi}{2} \right)^{1/2} P_{nm}(\xi) \exp(i m \theta) \]

3. 3-dimensional, spherical model (Hernquist & Ostriker 1992) : specialized for elliptical galaxy

\[ \rho_{nlm}(r) = \frac{K_{nl}}{2\pi} \frac{r^l}{r(1 + r)^{2l+3}} C_n^{(2l+3/2)}(\xi) \sqrt{4\pi} Y_{lm}(\theta, \phi) \]

\[ \Phi_{nlm}(r) = -\frac{r^l}{(1 + r)^{2l+1}} C_n^{(2l+3/2)}(\xi) \sqrt{4\pi} Y_{lm}(\theta, \phi) \]
Decomposition of Particles as Disk/Bulge Components

Fitting the surface & volume density profiles of the galaxy using three exponential disks

-> the most extended exponential component = disk
Decomposition of Particles as Disk/Bulge Components

<2-dimensional, cylindrical component>

<3-dimensional, spherical component>
Multipole Expansion for the Bulge Component

Spherical term (Hernquist & Ostriker 1992)

- using basis function of the density-potential pair of Hernquist (1990)

\[ \rho(r) = \frac{M \ a}{2\pi \ r \ (r + a)^3}, \quad \Phi(r) = -\frac{GM}{r + a} \]

-> a basis set

\[ \rho(r) = \sum_{n,m} A_{n,m} \rho_{n,m}(r), \quad \text{where} \quad \rho_{n,m}(r) = \frac{K_{nl}}{2\pi} \frac{r^l}{r (1 + r)^{2l+3}} \cdot C_n^{(2l+3/2)}(\xi) \sqrt{4\pi} \ Y_{lm}(\theta, \phi) \]

\[ \Phi(r) = \sum_{n,m} A_{n,m} \Phi_{n,m}(r), \quad \text{where} \quad \Phi_{n,m}(r) = -\frac{r^l}{(1 + r)^{2l+1}} \cdot C_n^{(2l+3/2)}(\xi) \sqrt{4\pi} \ Y_{lm}(\theta, \phi) \]

-> rewrite after some algebra

\[ \rho(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_{lm}(\cos \theta)[A_{lm}(r) \cos m\phi + B_{lm}(r) \sin m\phi] \]

\[ \Phi(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_{lm}(\cos \theta)[C_{lm}(r) \cos m\phi + D_{lm}(r) \sin m\phi] \]
Multipole Expansion for the Bulge Component

Spherical term (Hernquist & Ostriker 1992)

\[
\rho(r) = \frac{M}{2\pi} \frac{a}{r (r + a)^3}, \quad \phi(r) = -\frac{GM}{r + a}
\]

-> a basis set

\[
\rho(r) = \sum_{n,m} A_{nm} \rho_{nm}(r), \quad \text{where} \quad \rho_{nm}(r) = \frac{K_{nl}}{2\pi} \frac{r^l}{r(1 + r)^{2l+3}} \ C_n^{(2l+3/2)}(\xi) \sqrt{4\pi} \ Y_{lm}(\theta, \phi)
\]

\[
\Phi(r) = \sum_{n,m} A_{nm} \Phi_{nm}(r), \quad \text{where} \quad \Phi_{nm}(r) = -\frac{r^l}{(1 + r)^{2l+1}} \ C_n^{(2l+3/2)}(\xi) \sqrt{4\pi} \ Y_{lm}(\theta, \phi)
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-> rewrite after some algebra

\[
\rho(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_{lm}(\cos \theta) [A_{lm}(r) \cos m\phi + B_{lm}(r) \sin m\phi]
\]

\[
\Phi(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_{lm}(\cos \theta) [C_{lm}(r) \cos m\phi + D_{lm}(r) \sin m\phi]
\]

-> to get the coefficients using particle information
Multipole Expansion for the Bulge Component

\[- \text{to get acceleration}\]

\[
a_{t}(r, \theta, \phi) = - \sum_{l=0}^{\infty} \sum_{m=0}^{l} P_{lm}(\cos \theta) [E_{lm}(r) \cos m\phi + F_{lm}(r) \sin m\phi],
\]

\[
a_{\theta}(r, \theta, \phi) = - \frac{1}{r} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{dP_{lm}(\cos \theta)}{d\theta} [C_{lm}(r) \cos m\phi + D_{lm}(r) \sin m\phi],
\]

\[
a_{\phi}(r, \theta, \phi) = - \frac{1}{r} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \frac{mP_{lm}(\cos \theta)}{\sin \theta} [D_{lm}(r) \cos m\phi - C_{lm}(r) \sin m\phi].
\]

\[- \text{to get the coefficients}\]

\[
\begin{bmatrix}
E_{lm}(r) \\
F_{lm}(r)
\end{bmatrix} = N_{lm} \sum_{n=0}^{\infty} A_{nl} \frac{d}{dr} \tilde{\Phi}_{nl}(r) \sum_{k} m_{k} \tilde{\Phi}_{k}(r) P_{lm}(\cos \theta_{k}) \left( \begin{array}{c}
\cos m\phi_{k} \\
\sin m\phi_{k}
\end{array} \right)
\]

where

\[
\tilde{A}_{nl} \tilde{P}_{nl}(r) = - \frac{(n+2l+3/2)}{\pi^{1/2}} \frac{C_{n}^{(2l+3/2)}(\xi)}{K_{nl}} \left[ \frac{\Pi_{2l+2}^{4l+2}}{\Pi_{n+1}^{4l+2}} \right]^{2} \frac{r^{l-1}}{(1+r)^{2l+3}}
\]

\[
\tilde{A}_{nl} \tilde{\Phi}_{nl}(r) = 2\pi^{1/2} (n+2l+3/2) \frac{C_{n}^{(2l+3/2)}(\xi)}{K_{nl}} \left[ \frac{\Pi_{2l+2}^{4l+2}}{\Pi_{n+1}^{4l+2}} \right] \frac{r^{l}}{(1+r)^{2l+1}}
\]

\[
\tilde{A}_{nl} \frac{d}{dr} \tilde{\Phi}_{nl}(r) = \tilde{A}_{nl} \tilde{\Phi}_{nl}(r) \left[ \frac{l}{r} \frac{2l+1}{1+r} + \frac{4(2l+3/2)}{(1+r)^{2}} \frac{C_{n-1}^{(2l+5/2)}(\xi)}{C_{n-1}^{(2l+3/2)}(\xi)} \right]
\]
Multipole Expansion for the Bulge Component

<Density map from particle distribution>

<Density map using multipole expansion>
Multipole Expansion for the Disk Component

2-dimensional cylindrical term (Aoki & Iye 1978)

\[ \mu(R, \phi) = \sum_{n,m} A_{nm} \mu_{nm}(R, \phi) \]
\[ \Phi(R, \phi) = \sum_{n,m} A_{nm} \Phi_{nm}(R, \phi) \]

where
\[ \mu_{nm}(R, \phi) = \frac{Ma}{R_a^2} \frac{(2n+1)}{2\pi} P_{nm}(\xi) \exp(i m \phi) \]
\[ \Phi_{nm}(R, \phi) = -\frac{GM}{R_a} P_{nm}(\xi) \exp(i m \phi) \]

where
\[ \xi \equiv \frac{R^2 - a^2}{R^2 + a^2} \]

\[ \mu(R, \phi) = \frac{Ma}{R_a^2} \sum_{n,m} \varepsilon_m \frac{2n+1}{\pi} P_{nm}(\xi) \left( \frac{n-m}{n+m} \right)! \left[ I_{\text{c}} \cos(m \phi) + I_{\text{s}} \sin(m \phi) \right] \]
\[ \Phi(R, \phi) = -\frac{GM}{R_a} \sum_{n,m} \varepsilon_m P_{nm}(\xi) \left( \frac{2(n-m)}{(n+m)} \right)! \left[ I_{\text{c}} \cos(m \phi) + I_{\text{s}} \sin(m \phi) \right] \]

\[ I_{\text{c},nm} = \frac{a}{M} \int \frac{\mu(R', \phi')}{R_a^2} P_{nm}(\xi') \cos(m \phi') R' d\phi' dR' \]
\[ I_{\text{s},nm} = \frac{a}{M} \int \frac{\mu(R', \phi')}{R_a^2} P_{nm}(\xi') \sin(m \phi') R' d\phi' dR' \]

\[ a_R(R, \phi) = \frac{GMR}{R_a^3} \sum_{n,m} \varepsilon_m \left[ \frac{4a^2}{R_a^2} \frac{dP_{nm}(\xi)}{d\xi} - P_{nm}(\xi) \right] \frac{2(n-m)!}{(n+m)!} \left[ I_{\text{c}} \cos(m \phi) + I_{\text{s}} \sin(m \phi) \right] \]
\[ a_\phi(R, \phi) = \frac{GM}{R_a R} \sum_{n,m} \varepsilon_m P_{nm}(\xi) \frac{2(n-m)!}{(n+m)!} \frac{m}{m} \left[ I_{\text{c}} \cos(m \phi) - I_{\text{s}} \sin(m \phi) \right] \]
Multipole Expansion for the Disk Component

<Density map from particle distribution>

<Density map using multipole expansion>
Thick Disk Approximation

We need to correct $a_R$ and also assign $a_z$ caused by the disk component.

: by thick disk approximation (Binney & Tremaine 2008)

**2D razor-thin disk**

$$\Phi(R, 0) = -4G\Sigma_0 \int_0^R da \frac{aK_1(a/R_d)}{\sqrt{R^2 - a^2}} = -\pi G\Sigma_0 R [I_0(y)K_1(y) - I_1(y)K_0(y)]$$

where $K_0, K_1, I_0, I_1$ are Bessel functions, and

$$y \equiv \frac{R}{2R_d}$$

**3D exponential thick disk**

$$\Phi(R, z) = -\frac{4G\Sigma_0}{R_d} \int_{-\infty}^{\infty} d z' \zeta(z') \int_0^\infty da \sin^{-1}\left(\frac{2a}{\sqrt{+} + \sqrt{-}}\right) aK_0(a/R_d)$$

where $\sqrt{\pm} \equiv \sqrt{z^2 + (a \pm R)^2}$

$$\rho(R, z) = \Sigma(R)\zeta(z)$$
Acceleration Profiles

![Graphs showing acceleration profiles at different Z/kpc values.](image-url)
Hydrodynamics in the Realistic Galactic Potential

Based on Gadget-2 (Springel 2005)

- gravitational **accelerations by the galactic potentials using multipole expansions**
- radiative cooling for a gas with a solar metallicity for a range of $10 < T/K < 10^8$
  
  & uniform heating from far-ultraviolet radiation of 100 Habing field (Habing 1968)
  
  : using CLOUDY 90 package (Ferland et al. 1998)
- star formation as a single stellar population of Kroupa mass function (Kroupa 2001)
- supernovae feedback following stellar evolutionary model of Hurley, Pols & Tout (2000)

**Advantages of using multipole expansions**

- calculation time is **40~50 times faster** than live particles
- easy to get acceleration for any shapes of the galactic potential
- easy to change elongation, orientation, total mass, scale, etc.
- able to describe evolution of the realistic galactic potential as time
Test Run – without spiral arms

In case of a snapshot without spiral arms + pattern speed of 45 km/s/kpc

10^5 gas particles following exponential density profile

Initial location of gas particles = 0~5kpc

Initial location of gas particles = 1~5kpc
Bar strength measured by Q parameter of $Q_b \equiv \frac{F_T(r, \phi)}{F_R(r)} \bigg|_{\text{max}}$

Higher Q value: increasing expansion coefficient of $m=2$ of the axi-symmetric bulge

[Images showing density profiles for $Q=\sim0.1$ and $Q=\sim0.2$]
Test Run – without spiral arms

Bar strength measured by Q parameter of

$$Q_b \equiv \frac{F_T(r, \phi)}{F_R(r)} \bigg|_{\text{max}}$$

Higher Q value: increasing expansion coefficient of m=2 of the axi-symmetric bulge
Test Run – with spiral arms

Q=~0.1

Q=~0.2
Test Run – star formation

Most of gas particles are distributed near $\sim 10^4$ K or equilibrium temperature. Stars are formed when $n_H > 100 \text{ cm}^{-3}$, $T < 100 \text{ K}$, $\nabla \cdot \mathbf{v} < 0$. 

\[ n_H = 100 \text{ cm}^{-3} \]
Future Work

Formation/evolution of nuclear ring
- by different bar strength and/or existence of the spiral structures
- origin of gas in the nuclear region (gas inflow from the galactic disk)
- further inward migration of gas from CMZ to CND
- star formation rate, star formation history in the nuclear region
- nuclear bulge growth

With more realistic galactic potential
- series of simulation snapshot, interpolating expansion coefficients
- nested bar, fluctuating motions in the CMZ, twisted nuclear ring, etc.