# Formation of Nuclear Ring from Hydrodynamic Simulations using Multipole Expansions

**Preliminary result**

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# Development of Hydrodynamic code - 1

To simulate the sub-galactic scale structure formation in the Lambda CDM model,

we have developed a new cosmological hydrodynamic code. (Shin, Kim, Kim, & Park 2014)

-> **EU$\Lambda$HA** : **E**volution of the **U**niverse simulated with ***N***-body and **H**ydrodynamic **A**lgorithms

- based on the most efficient code (PM+tree) for the large scale structure, GOTPM (Dubinsky, Kim, & Park 2003)
- improved the hydrodynamics (SPH) into the GOTPM code (mainly by Juhan Kim)
- used a time-step limiter scheme (Saitoh & Makino 2009) over the individual time step
- added the realistic baryonic physics

: Reionization process by UV sources and UV shielding (Haardt & Madau 1996, Sawala et al. 2010)

: Radiative heating/cooling (T ~ reach to 100K) using CLOUDY 90 package (Ferland et al. 1998)

: Star formation as single stellar population (Katz 1992, Abadi et al. 2003, Kroupa 2001)

: Metal and energy feedback by $SN_{II}$ (Hurley, Pols, Tout 2000, Okamoto, Nemmen, & Bower 2008, Woosley & Weaver 1995)

# Development of Hydrodynamic code - 2

To simulate the sub-galactic structure formation/evolution in non-cosmological frame,

we put the astrophysical gas processes (Shin, Kim, Kim, & Park 2014) into GADGET-2 (Springel 2005).

- radiative heating/cooling, reionization, shielding, star formation, and SN feedback
- mainly by Kyungwon Chun (see poster, P-1)

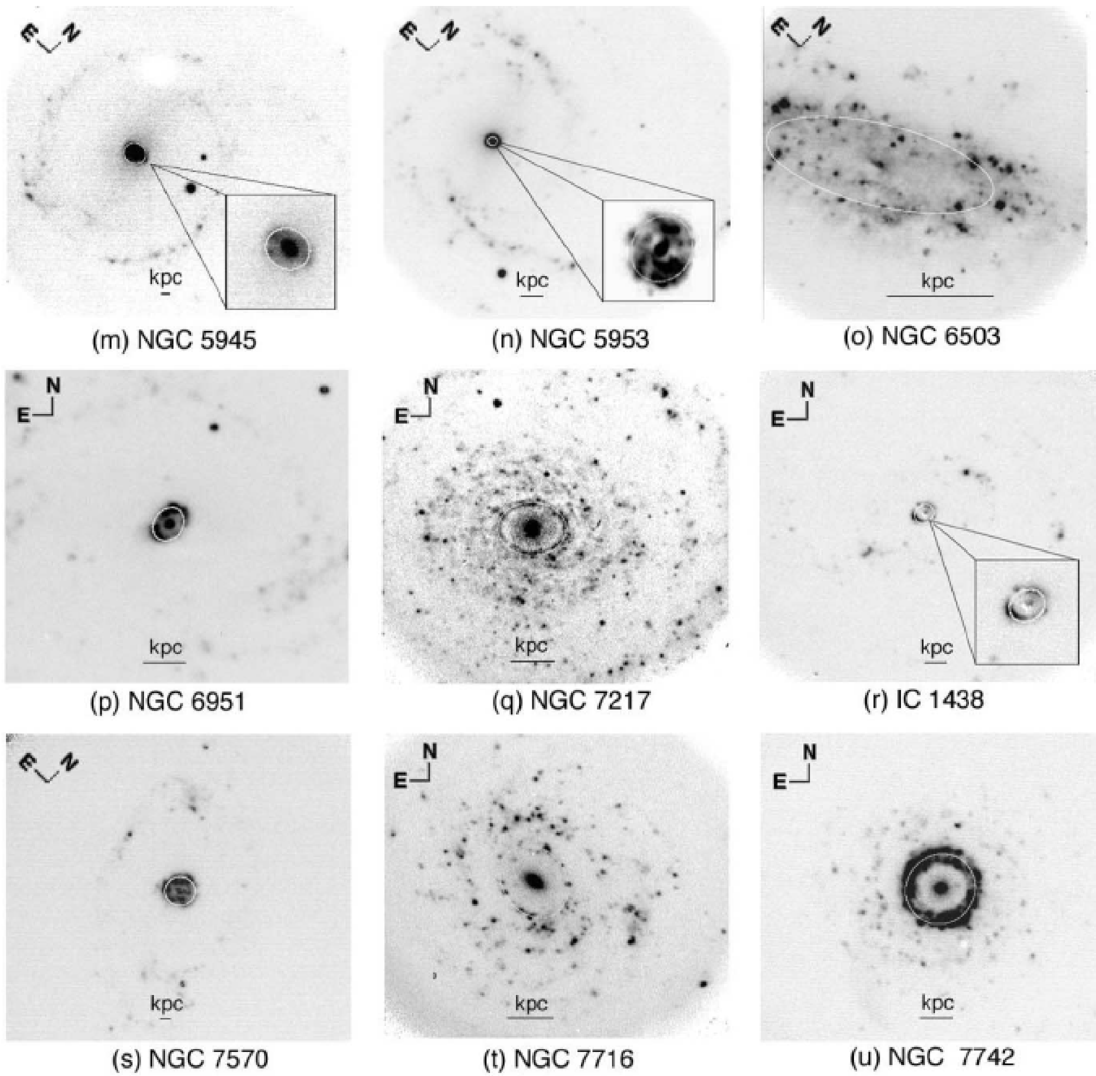
Research plans

- formation/evolution of CMZ (central molecular zone), nuclear bulge, and CND (circumnuclear disk)
- formation of sub-galactic structures (globular clusters, ultra compact dwarf galaxies) in cosmological frame

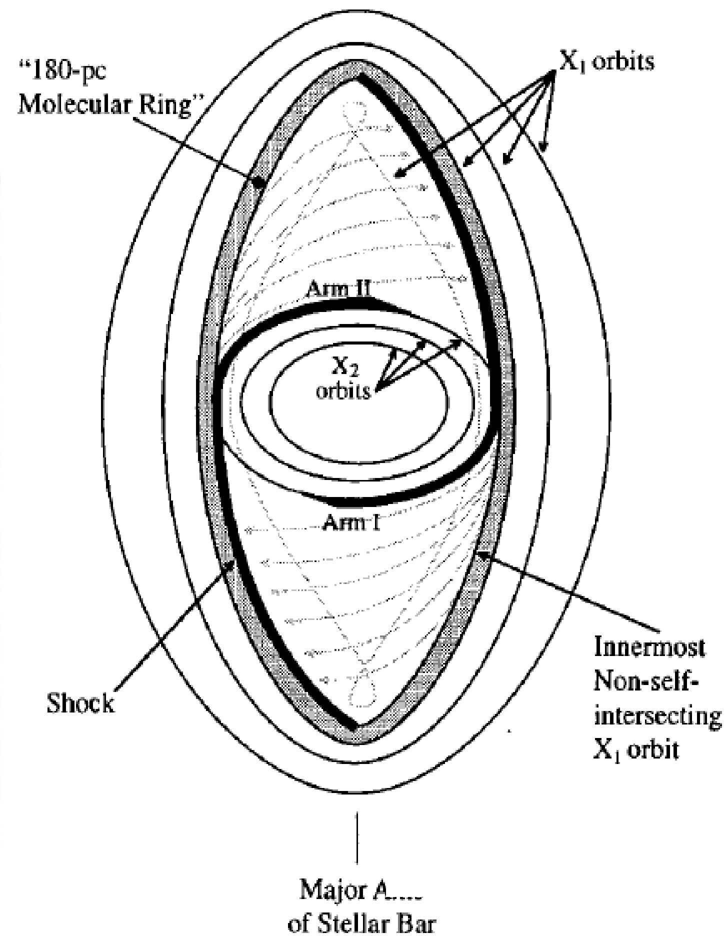
Code development plans

- based on Gadget-3 for better scalability
- including multiphase model
- including different star formation efficiency for different gas density (Kruijssen 2012)

# Nuclear Star-Forming Rings in External Galaxies

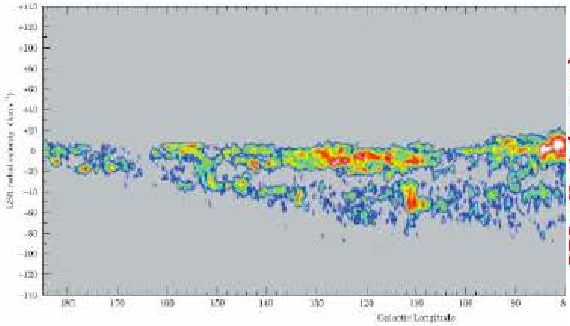


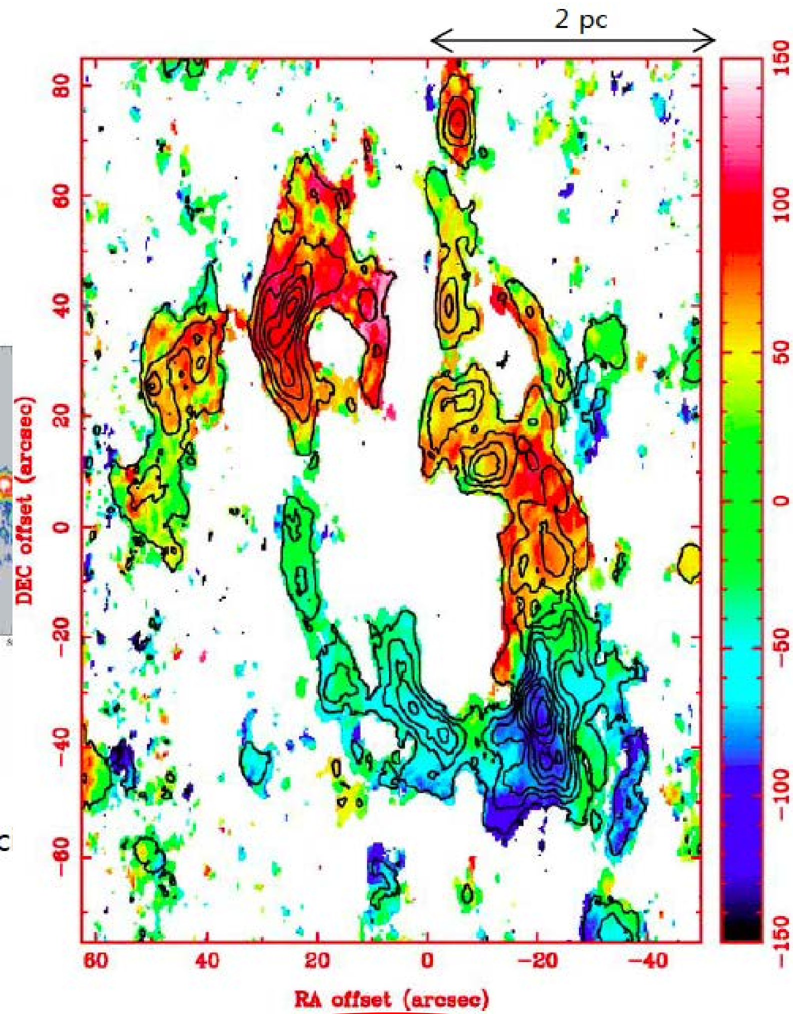

Mazzuca et al. 2008

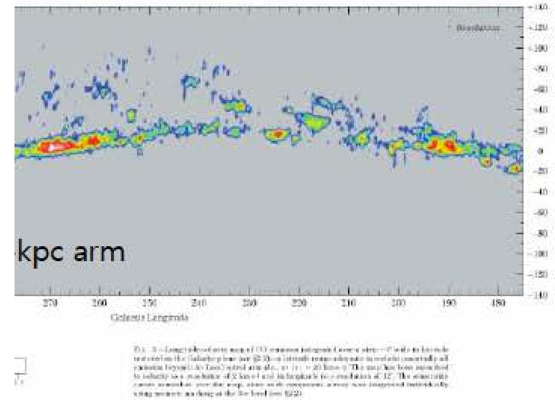



Morris & Serabyn 1996

# The Milky Way also has a Bar




lecular Zone

r ring at ~4 kpc

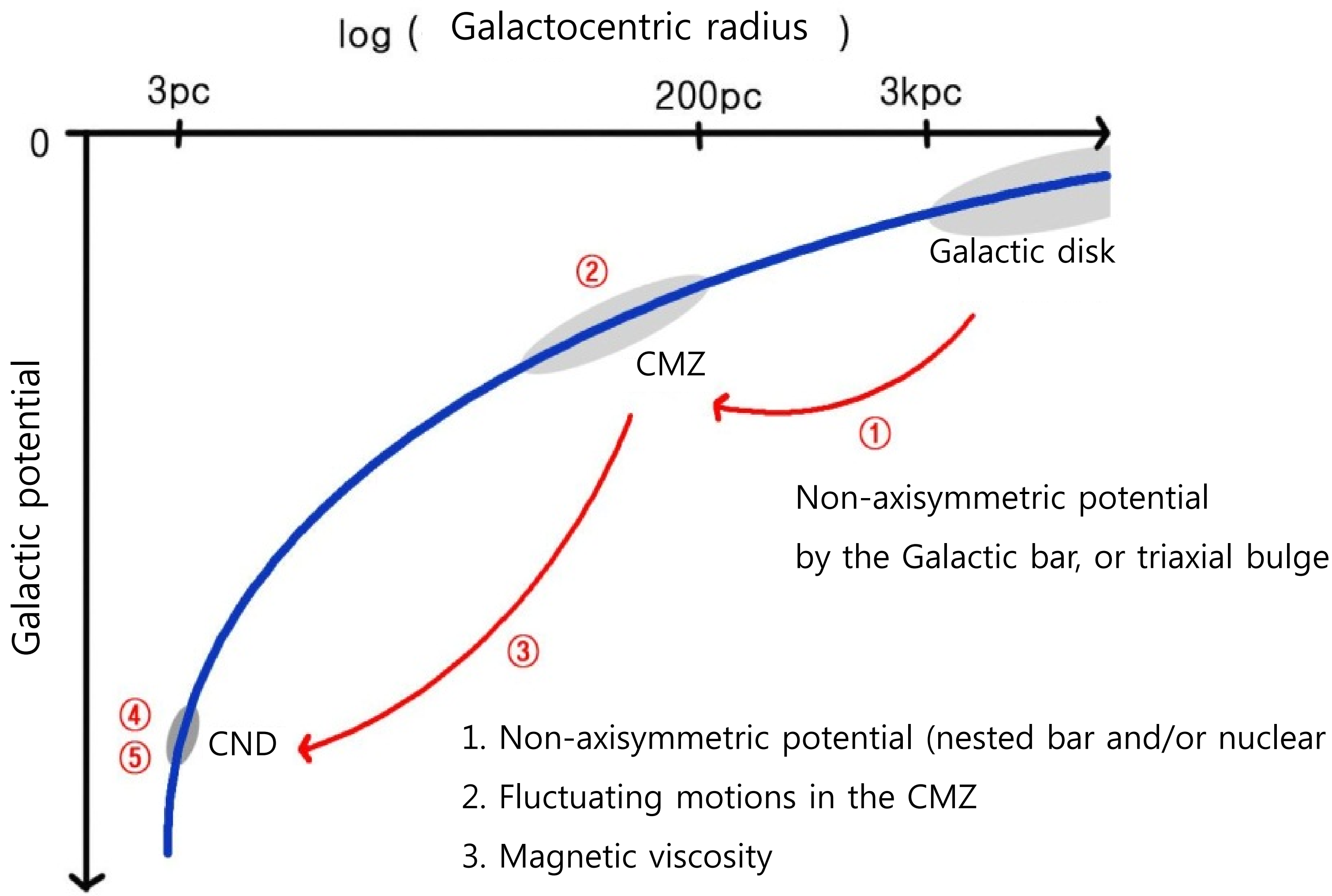
kpc arm

Longitude-veloc

ann & Thaddeus, 2001).

Circumnuclear Disk

# Migration of Gas toward the Galactic Center




1. Non-axisymmetric potential (nested bar and/or nuclear bulge)
2. Fluctuating motions in the CMZ
3. Magnetic viscosity
4. Dynamical friction

# Recent Numerical Simulations

Kim, S. S., Saitoh, T., Jeon, M., Figer, D., Merritt, D., & Wada, K., 2011

-> 3D hydrodynamic simulation using particle-based code (ASURA, Saitoh et al. 2008, 2009)

+ including radiative cooling, star formation, SN feedback

+ galactic potential : m=2 bar with a power-law density profile for the inner Galactic bulge

Kim, W.-T. & Seo, W.-Y., & Kim, Y., 2012

-> 2D hydrodynamic simulation using grid-based code (CMHOG, Kim, W.-T. et al. 2012)

+ galactic potential :non-axysymmetric stellar bar (Ferrers 1987), stellar disk, stellar bulge, central BH

+ formation and evolution of gaseous substructures in barred galaxies with varying bar strength

Seo, W.-Y., & Kim, Y.-T., 2013

-> Based on Kim, W.-T. & Seo, W.-Y., & Kim, Y., 2012

+ including star formation and SN feedback

Seo, W.-Y., & Kim, Y.-Y., in press

-> based on Seo, W.-Y., & Kim, Y.-T., 2013

+ galactic potential : spiral arms

# Our Strategy on Numerical Simulations

3-dimensional hydrodynamic simulation

+ realistic baryonic physics (including radiative cooling/heating, star formation, SN feedback)

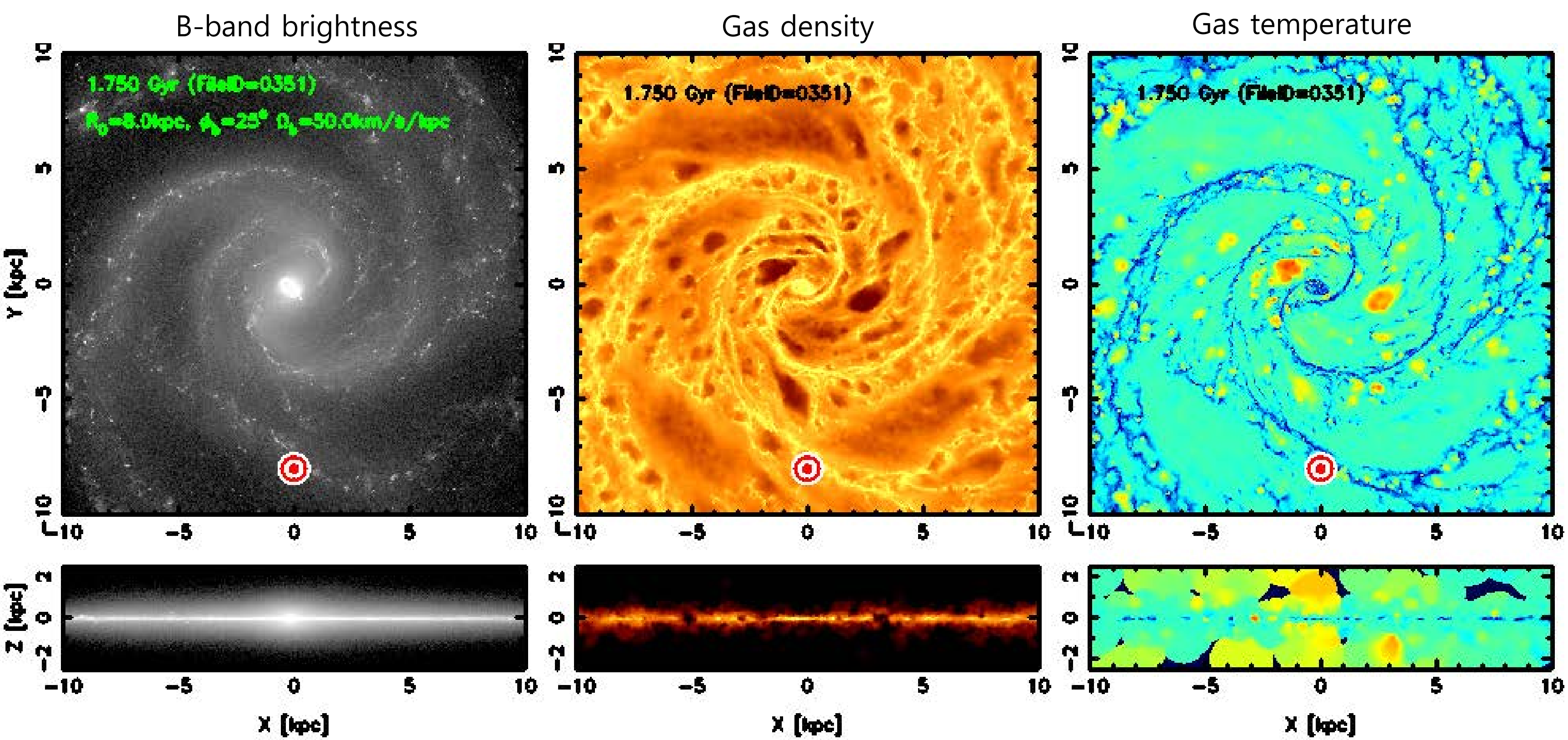
+ realistic mass inflow toward the Galactic center from the disk

+ long term evolution to see formation of the nuclear bulge

+ realistic galactic potential of the Galaxy

How do we consider the realistic galactic potential of the Milky Way?

# Modeling the Galactic Potentials using Multipole Expansions




Simulation snapshots for the Milky Way-like galaxy model (of J. Baba & T. Saitoh)

( ~$10^7$ gas and star particles for both disk and bulge components + fixed halo model)

+ multipole expansions for modeling the realistic galactic potentials

# Previous Numerical Studies with Multipole Expansions

1. 2-dimensional, cylindrical model (Aoki & Iye 1978) : disk only system

$$\mu_{nm}(R,\phi) = \frac{Ma}{R_a^3}\frac{(2n+1)}{2\pi}P_{nm}(\xi)\exp(im\phi)$$

$$\Phi_{nm}(R,\phi) = -\frac{GM}{R_a}P_{nm}(\xi)\exp(im\phi)$$

2. 3-dimensional, cylindrical model (Hozumi & Hernquist 2005) : numerical problems

$$\mu_{nm}(R) = \frac{2n+1}{2\pi}\left(\frac{1-\xi}{2}\right)^{3/2}P_{nm}(\xi)\exp(im\theta)$$

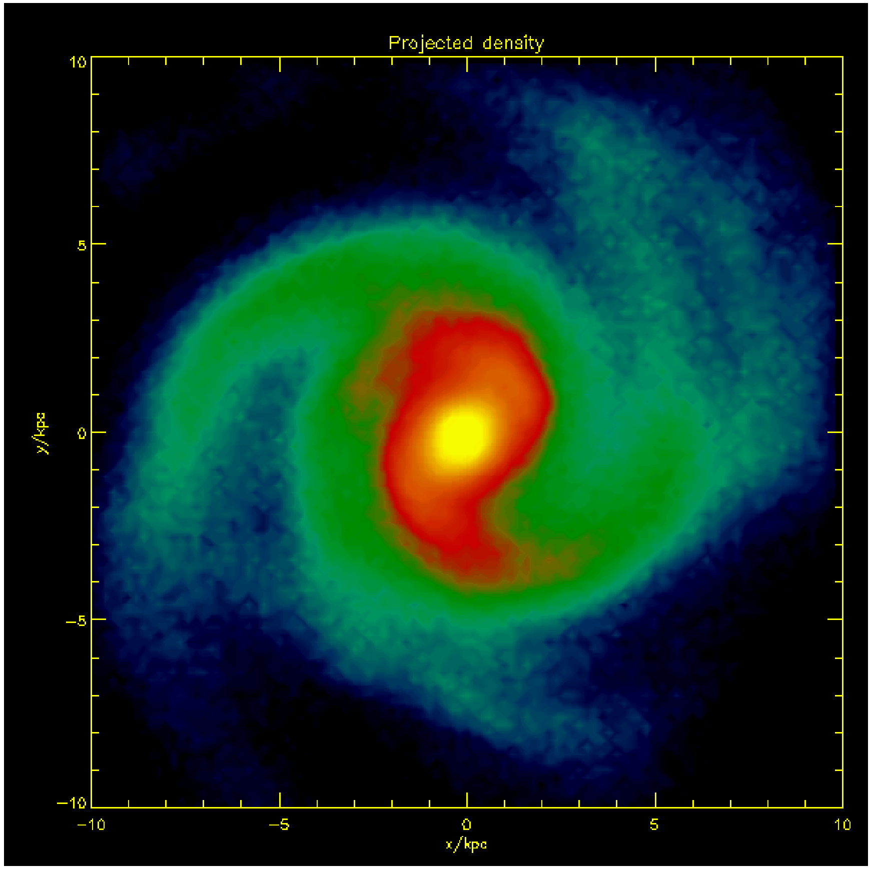
$$\Phi_{nm}(R) = -\left(\frac{1-\xi}{2}\right)^{1/2}P_{nm}(\xi)\exp(im\theta),$$

3. 3-dimensional, spherical model (Hernquist & Ostriker 1992) : specialized for elliptical galaxy

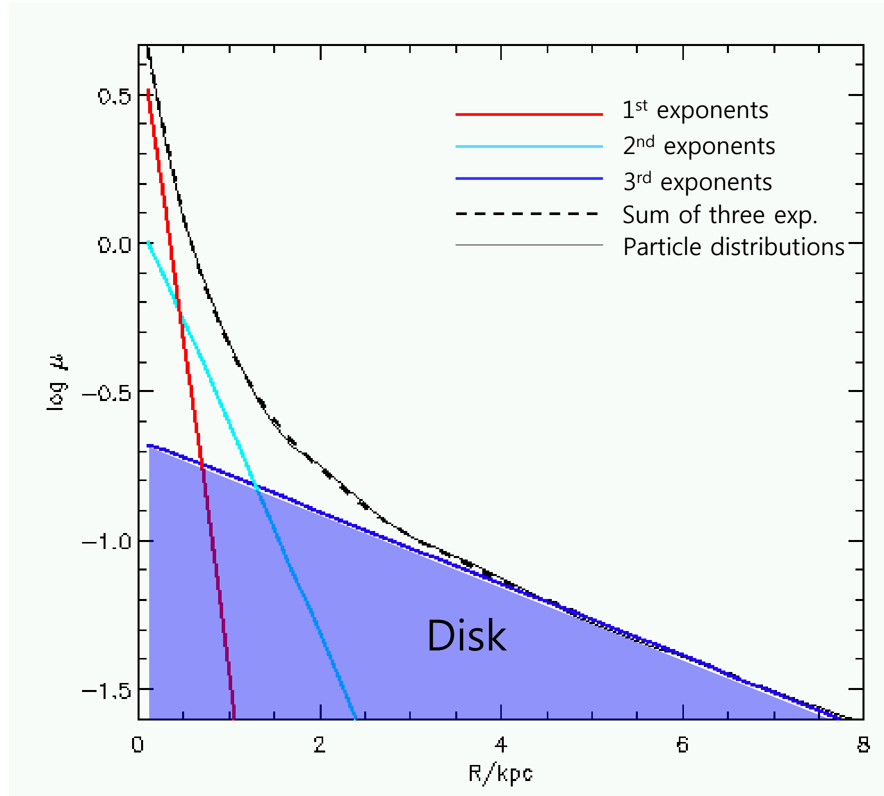
$$\rho_{nlm}(r) = \frac{K_{nl}}{2\pi}\frac{r^l}{r(1+r)^{2l+3}}C_n^{(2l+3/2)}(\xi)\sqrt{4\pi}\,Y_{lm}(\theta,\phi)$$

$$\Phi_{nlm}(r) = -\frac{r^l}{(1+r)^{2l+1}}C_n^{(2l+3/2)}(\xi)\sqrt{4\pi}\,Y_{lm}(\theta,\phi)$$

# Decomposition of Particles as Disk/Bulge Components




<Density map from a simulation snapshot>

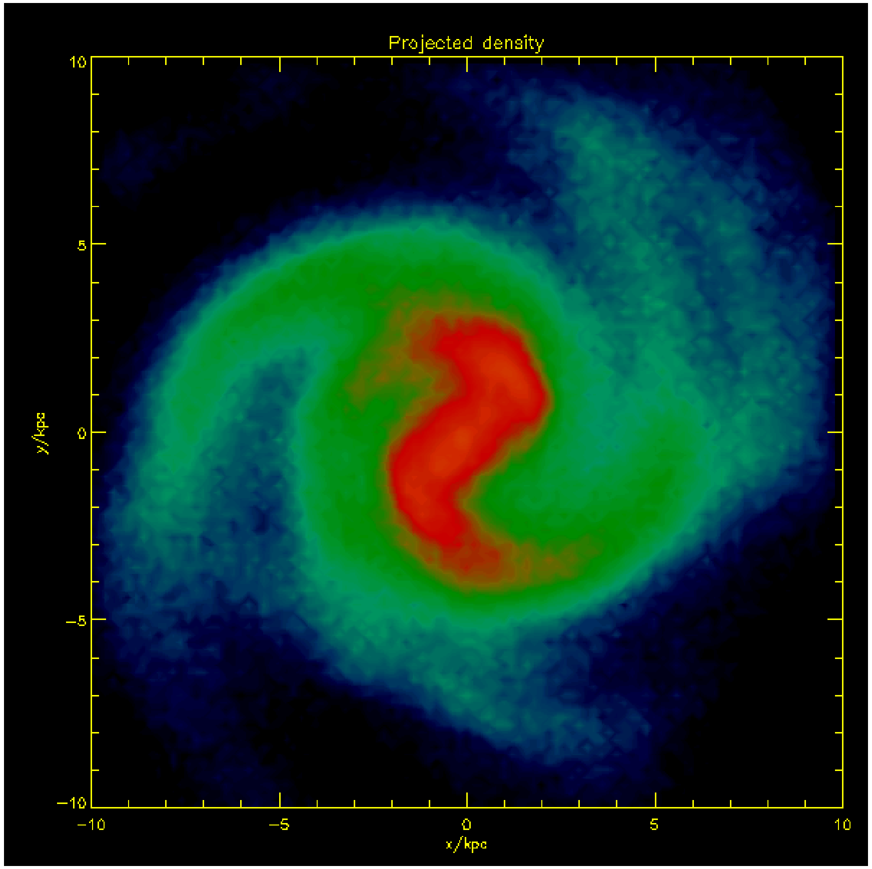



<Surface density profile>

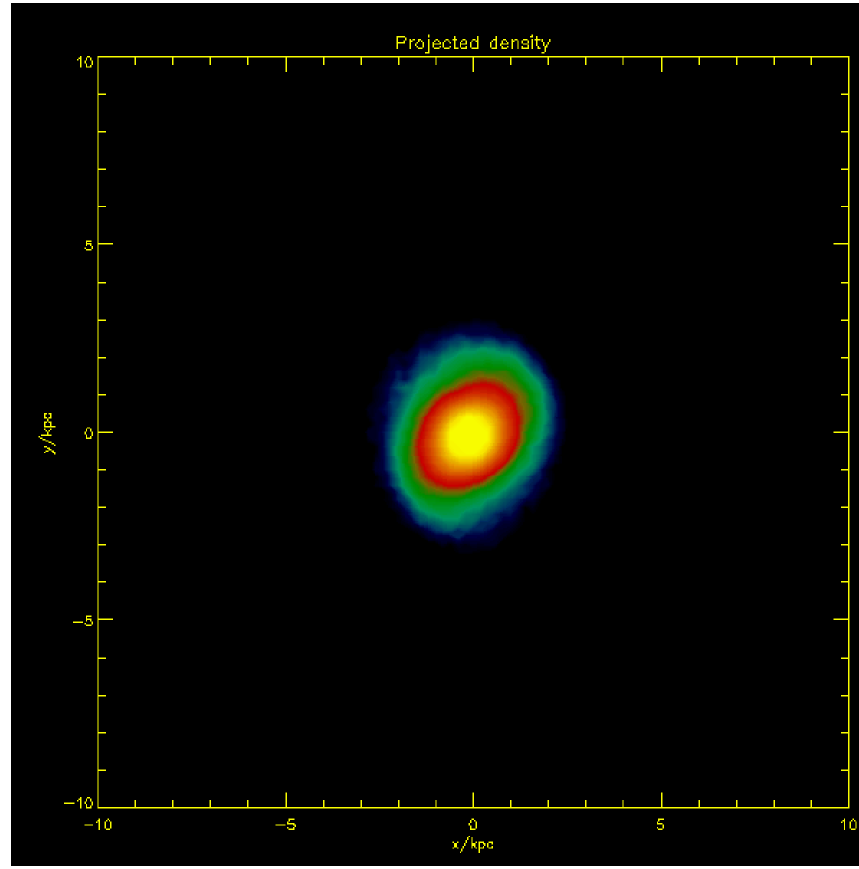
Fitting the surface & volume density profiles of the galaxy using three exponential disks

-> the most extended exponential component = disk

# Decomposition of Particles as Disk/Bulge Components




<2-dimensional, cylindrical component>




<3-dimensional, spherical component>

# Multipole Expansion for the Bulge Component

Spherical term (Hernquist & Ostriker 1992)

: using basis function of the density-potential pair of Hernquist (1990)

$$\rho(r) = \frac{M}{2\pi}\frac{a}{r}\frac{1}{(r+a)^3}, \qquad \varphi(r) = -\frac{GM}{r+a}$$

-> a basis set

Gegenbauer Polynomial

$$\rho(\mathbf{r}) = \sum_{nlm} A_{nlm}\rho_{nlm}(\mathbf{r}), \quad \text{where} \quad \rho_{nlm}(\mathbf{r}) = \frac{K_{nl}}{2\pi}\frac{r^l}{r(1+r)^{2l+3}} C_n^{(2l+3/2)}(\xi)\sqrt{4\pi}\, Y_{lm}(\theta, \phi)$$

$$\Phi(\mathbf{r}) = \sum_{nlm} A_{nlm}\Phi_{nlm}(\mathbf{r}), \quad \text{where} \quad \Phi_{nlm}(\mathbf{r}) = -\frac{r^l}{(1+r)^{2l+1}} C_n^{(2l+3/2)}(\xi)\sqrt{4\pi}\, Y_{lm}(\theta, \phi)$$

$$\xi = \frac{r-1}{r+1}$$

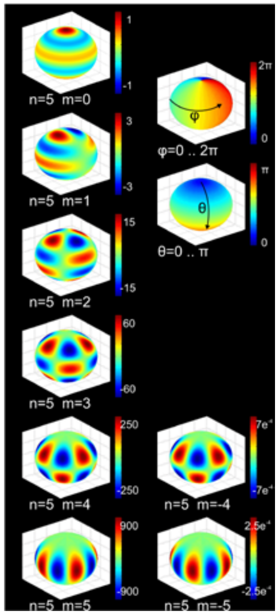
-> rewrite after some algebra

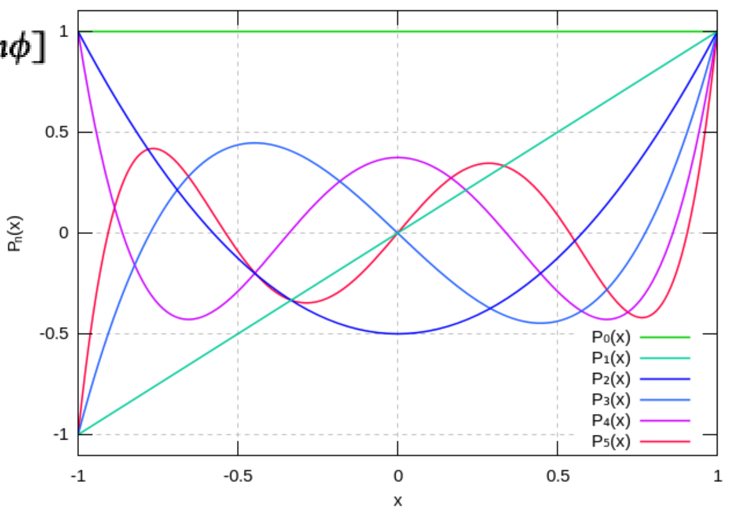
$$\rho(r, \theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\cos\theta)[A_{lm}(r)\cos m\phi + B_{lm}(r)\sin m\phi]$$

$$\Phi(r, \theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\cos\theta)[C_{lm}(r)\cos m\phi + D_{lm}(r)\sin m\phi]$$

Associated Legendre Polynomial

Spherical harmonic



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-> a basis set

$$\rho(\mathbf{r}) = \sum_{nlm} A_{nlm}\rho_{nlm}(\mathbf{r}), \quad \text{where} \quad \rho_{nlm}(\mathbf{r}) = \frac{K_{nl}}{2\pi}\frac{r^l}{r(1+r)^{2l+3}} C_n^{(2l+3/2)}(\xi)\sqrt{4\pi}\, Y_{lm}(\theta, \phi)$$

$$\Phi(\mathbf{r}) = \sum_{nlm} A_{nlm}\Phi_{nlm}(\mathbf{r}), \quad \text{where} \quad \Phi_{nlm}(\mathbf{r}) = -\frac{r^l}{(1+r)^{2l+1}} C_n^{(2l+3/2)}(\xi)\sqrt{4\pi}\, Y_{lm}(\theta, \phi)$$

-> rewrite after some algebra

$$\rho(r, \theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\cos\theta)[A_{lm}(r)\cos m\phi + B_{lm}(r)\sin m\phi]$$

$$\Phi(r, \theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\cos\theta)[C_{lm}(r)\cos m\phi + D_{lm}(r)\sin m\phi]$$

-> to get the coefficients using particle information

$$\begin{pmatrix} A_{lm}(r) \\ B_{lm}(r) \\ C_{lm}(r) \\ D_{lm}(r) \end{pmatrix} = \boxed{N_{lm}} \sum_{n=0}^{\infty} \tilde{A}_{nl} \begin{pmatrix} \tilde{\rho}_{nl}(r) \\ \tilde{\rho}_{nl}(r) \\ \tilde{\Phi}_{nl}(r) \\ \tilde{\Phi}_{nl}(r) \end{pmatrix} \boxed{\sum_k m_k \tilde{\Phi}_{nl}(r_k) P_{lm}(\cos\theta_k) \begin{pmatrix} \cos m\phi_k \\ \sin m\phi_k \\ \cos m\phi_k \\ \sin m\phi_k \end{pmatrix}}$$

# Multipole Expansion for the Bulge Component

-> to get acceleration

$$a_r(r,\theta,\phi) = -\sum_{l=0}^{\infty}\sum_{m=0}^{l} P_{lm}(\cos\theta)[E_{lm}(r)\cos m\phi + F_{lm}(r)\sin m\phi]\,,$$

$$a_\theta(r,\theta,\phi) = -\frac{1}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l} \frac{dP_{lm}(\cos\theta)}{d\theta}[C_{lm}(r)\cos m\phi + D_{lm}(r)\sin m\phi]\,,$$

$$a_\phi(r,\theta,\phi) = -\frac{1}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l} \frac{mP_{lm}(\cos\theta)}{\sin\theta}[D_{lm}(r)\cos m\phi - C_{lm}(r)\sin m\phi]\,,$$

-> to get the coefficients

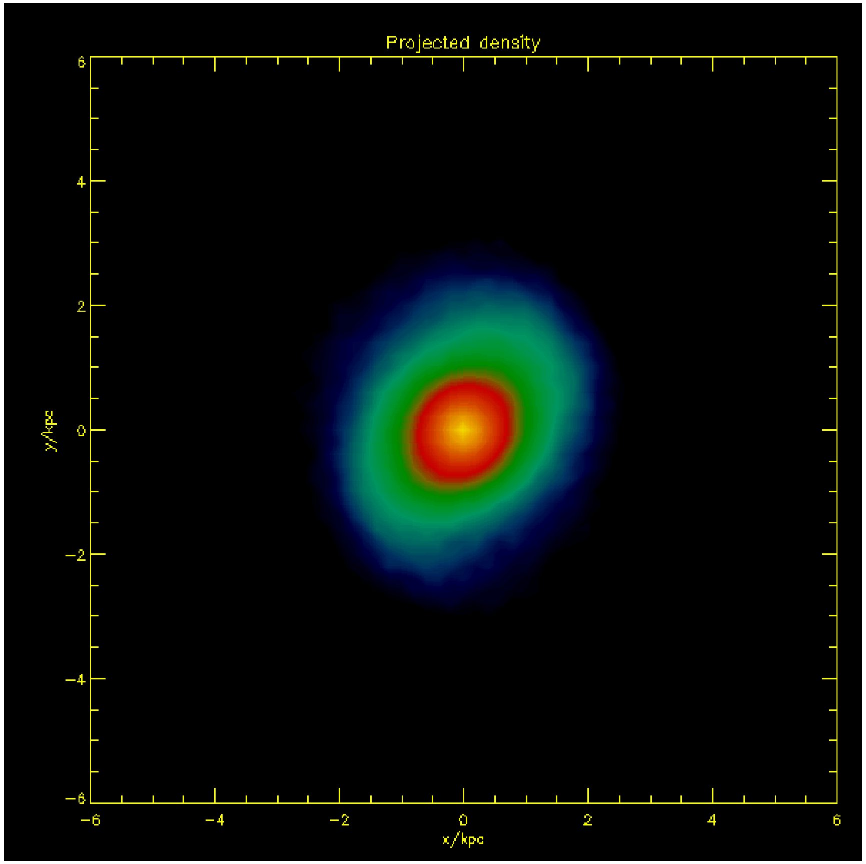
$$\begin{bmatrix} E_{lm}(r) \\ F_{lm}(r) \end{bmatrix} = \boxed{N_{lm}} \sum_{n=0}^{\infty} \tilde{A}_{nl} \frac{d}{dr}\tilde{\Phi}_{nl}(r) \boxed{\sum_k m_k \tilde{\Phi}_{nl}(r_k) P_{lm}(\cos\theta_k) \begin{pmatrix} \cos m\phi_k \\ \sin m\phi_k \end{pmatrix}}$$

where $$\tilde{A}_{nl}\,\tilde{\rho}_{nl}(r) = -\frac{(n+2l+3/2)}{\pi^{1/2}} C_n^{(2l+3/2)}(\xi) \frac{\left[\Pi_{2l+2}^{4l+2}\right]^2}{\Pi_{n+1}^{n+4l+2}} \frac{r^{l-1}}{(1+r)^{2l+3}}$$

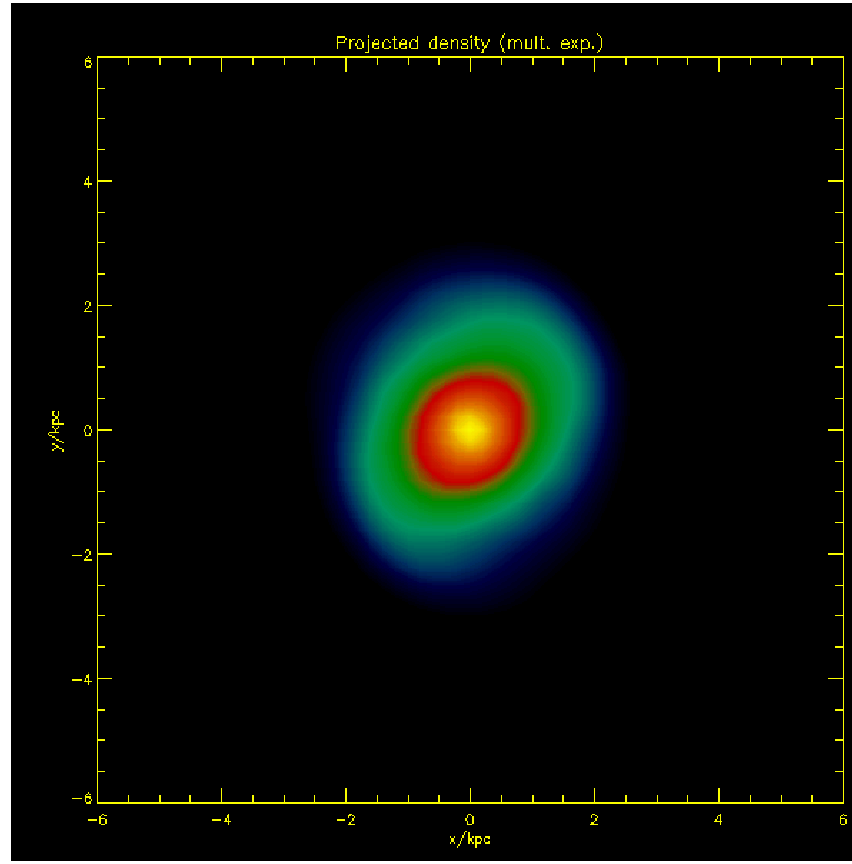
$$\tilde{A}_{nl}\,\tilde{\Phi}_{nl}(r) = 2\pi^{1/2}(n+2l+3/2)\frac{C_n^{(2l+3/2)}(\xi)}{K_{nl}} \frac{\left[\Pi_{2l+2}^{4l+2}\right]^2}{\Pi_{n+1}^{n+4l+2}} \frac{r^{l}}{(1+r)^{2l+1}}$$

$$\tilde{A}_{nl}\frac{d}{dr}\tilde{\Phi}_{nl}(r) = \tilde{A}_{nl}\,\tilde{\Phi}_{nl}(r)\left[\frac{l}{r} - \frac{2l+1}{1+r} + \frac{4(2l+3/2)}{(1+r)^2}\frac{C_{n-1}^{(2l+5/2)}(\xi)}{C_n^{(2l+3/2)}(\xi)}\right]$$

# Multipole Expansion for the Bulge Component




<Density map from particle distribution>




<Density map using multipole expansion>

# Multipole Expansion for the Disk Component

2-dimensional cylindrical term (Aoki & Iye 1978)

-> a basis set

$$\begin{cases} \mu(R,\phi) = \sum_{nm} A_{nm}\mu_{nm}(R,\phi) \\ \Phi(R,\phi) = \sum_{nm} A_{nm}\Phi_{nm}(R,\phi) \end{cases}$$

where

$$\begin{cases} \mu_{nm}(R,\phi) = \frac{Ma}{R_a^3}\frac{(2n+1)}{2\pi}P_{nm}(\xi)\exp(im\phi) \\ \Phi_{nm}(R,\phi) = -\frac{GM}{R_a}P_{nm}(\xi)\exp(im\phi) \end{cases}$$

where $\xi \equiv \frac{R^2 - a^2}{R^2 + a^2}$

->

$$\begin{cases} \mu(R,\phi) = \frac{Ma}{R_a^3}\sum_{n,m}\varepsilon_m \frac{2n+1}{\pi}P_{nm}(\xi)\frac{(n-m)!}{(n+m)!}[I_c\cos(m\phi)+I_s\sin(m\phi)] \\ \Phi(R,\phi) = -\frac{GM}{R_a}\sum_{n,m}\varepsilon_m P_{nm}(\xi)\frac{2(n-m)!}{(n+m)!}[I_c\cos(m\phi)+I_s\sin(m\phi)] \end{cases}$$

-> to get the coefficients

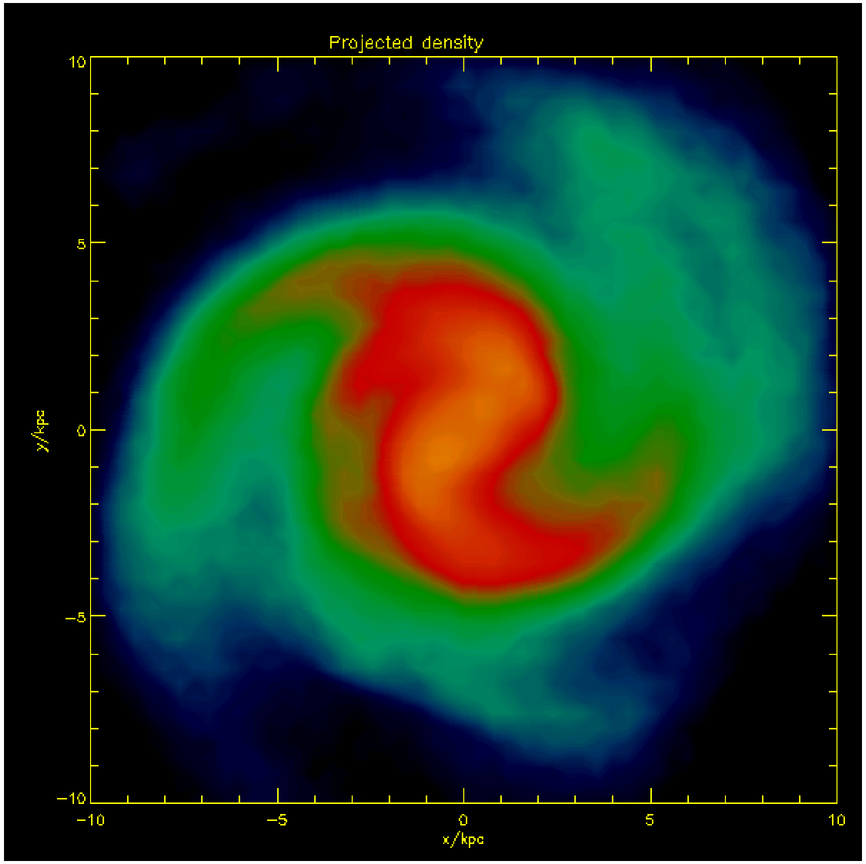
$$I_{c,nm} \equiv \frac{a}{M}\iint \frac{\mu(R',\phi')}{R'_a}P_{nm}(\xi')\cos(m\phi')R'd\phi'dR'$$

$$I_{s,nm} \equiv \frac{a}{M}\iint \frac{\mu(R',\phi')}{R'_a}P_{nm}(\xi')\sin(m\phi')R'd\phi'dR'$$

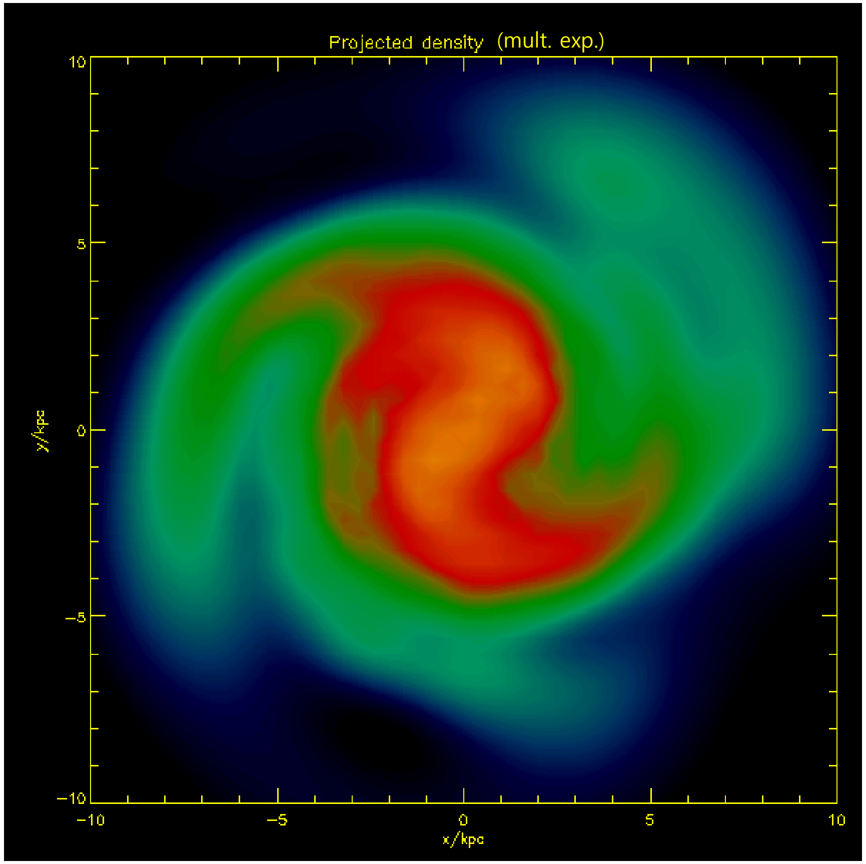
-> to get acceleration

$$\begin{cases} a_R(R,\phi) = \frac{GMR}{R_a^3}\sum_{n,m}\varepsilon_m\left[\frac{4a^2}{R_a^2}\frac{dP_{nm}(\xi)}{d\xi} - P_{nm}(\xi)\right]\frac{2(n-m)!}{(n+m)!}[I_c\cos(m\phi)+I_s\sin(m\phi)] \\ a_\phi(R,\phi) = \frac{GM}{R_aR}\sum_{n,m}\varepsilon_m P_{nm}(\xi)\frac{2(n-m)!}{(n+m)!}m[I_s\cos(m\phi)-I_c\sin(m\phi)] \end{cases}$$

# Multipole Expansion for the Disk Component




<Density map from particle distribution>




<Density map using multipole expansion>

# Thick Disk Approximation

We need to correct $a_R$ and also assign $a_z$ caused by the disk component.

: by thick disk approximation (Binney & Tremaine 2008)

## 2D razor-thin disk

$$\Phi(R,0) = -4G\Sigma_0 \int_0^R \mathrm{d}a \, \frac{aK_1(a/R_\mathrm{d})}{\sqrt{R^2 - a^2}} = -\pi G\Sigma_0 R\left[I_0(y)K_1(y) - I_1(y)K_0(y)\right]$$

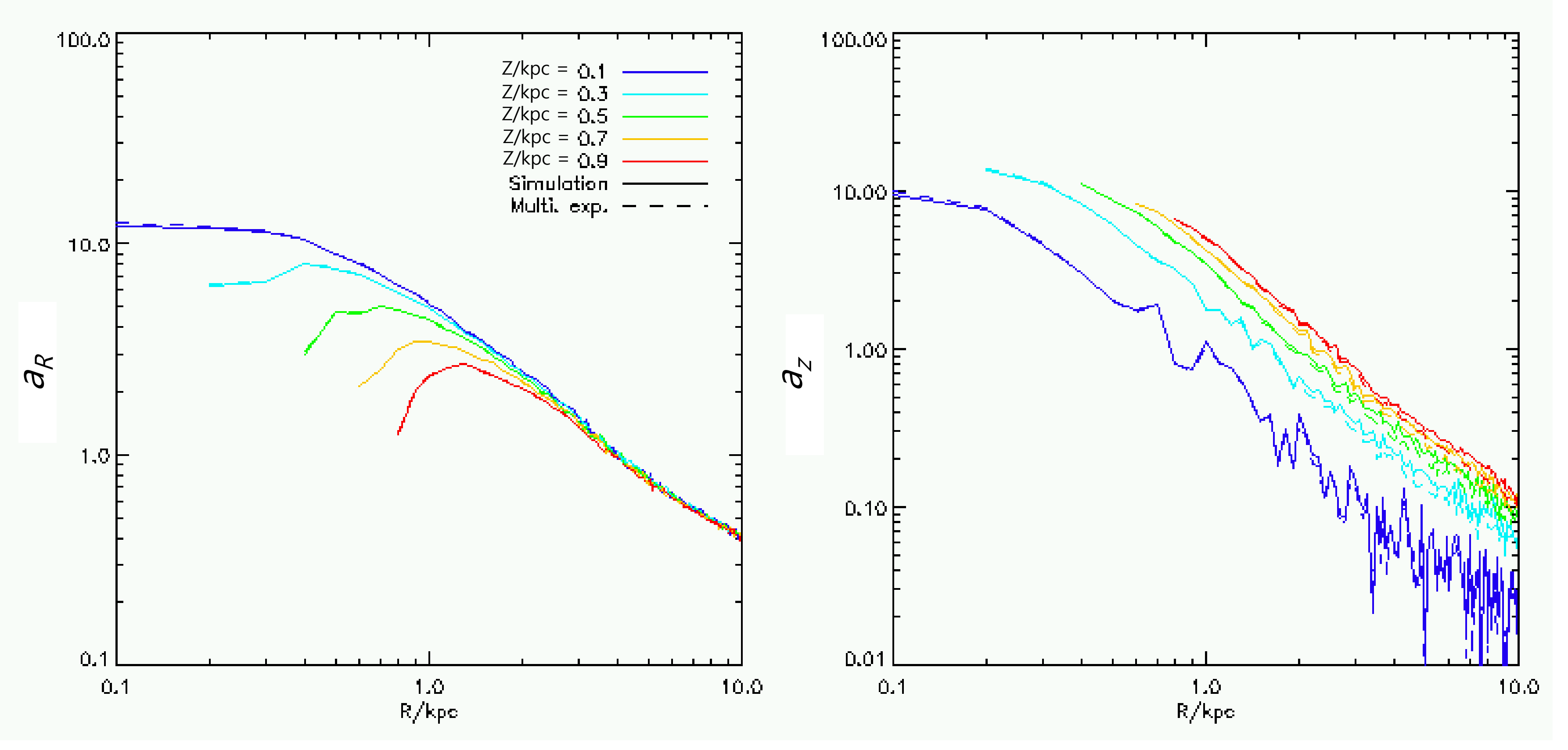
where $K_0$, $K_1$, $I_0$, $I_1$ are Bessel functions, and $y \equiv \frac{R}{2R_\mathrm{d}}$

## 3D exponential thick disk

$$\Phi(R,z) = -\frac{4G\Sigma_0}{R_\mathrm{d}} \int_{-\infty}^{\infty} \mathrm{d}z' \, \zeta(z') \int_0^{\infty} \mathrm{d}a \, \sin^{-1}\left(\frac{2a}{\sqrt{+} + \sqrt{-}}\right) aK_0(a/R_\mathrm{d})$$

where $\sqrt{\pm} \equiv \sqrt{z^2 + (a \pm R)^2}$ $\qquad \rho(R,z) = \Sigma(R)\zeta(z)$

# Acceleration Profiles



# Hydrodynamics in the Realistic Galactic Potential

Based on Gadget-2 (Springel 2005)

- gravitational accelerations by the galactic potentials using multipole expansions
- radiative cooling for a gas with a solar metallicity for a range of $10<T/K<10^8$

  & uniform heating from far-ultraviolet radiation of 100 Habing field (Habing 1968)

  : using CLOUDY 90 package (Ferland et al. 1998)
- star formation as a single stellar population of Kroupa mass function (Kroupa 2001)
- supernovae feedback following stellar evolutionary model of Hurley, Pols & Tout (2000)

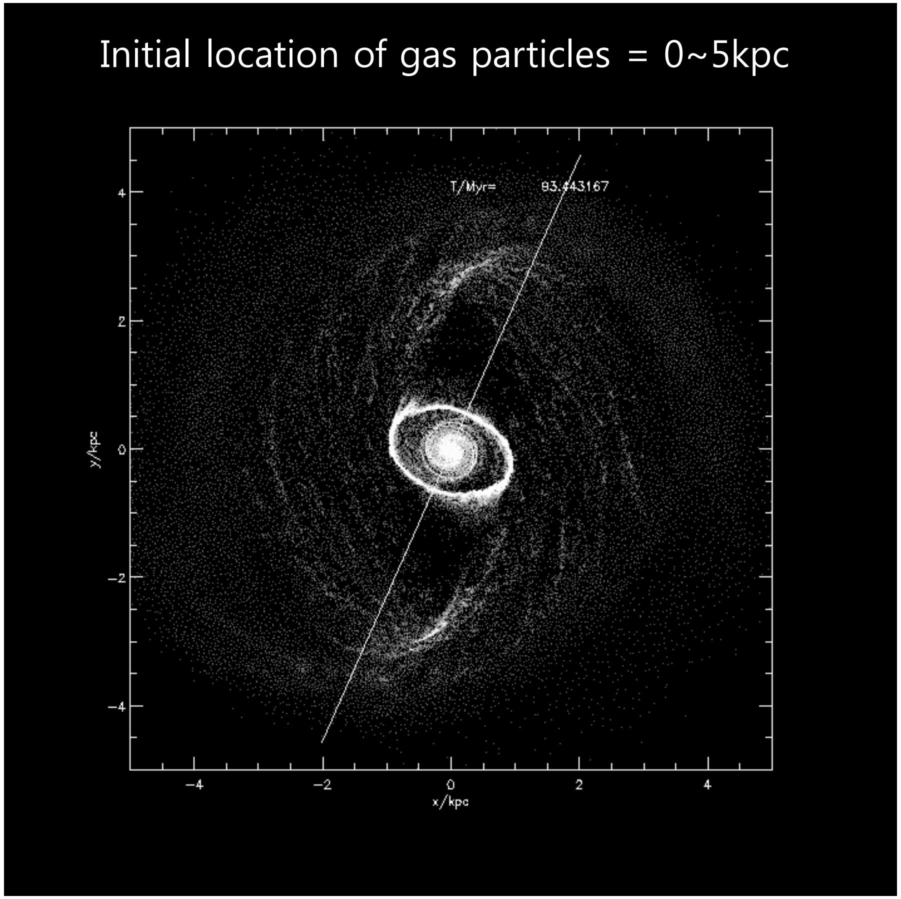
Advantages of using multipole expansions

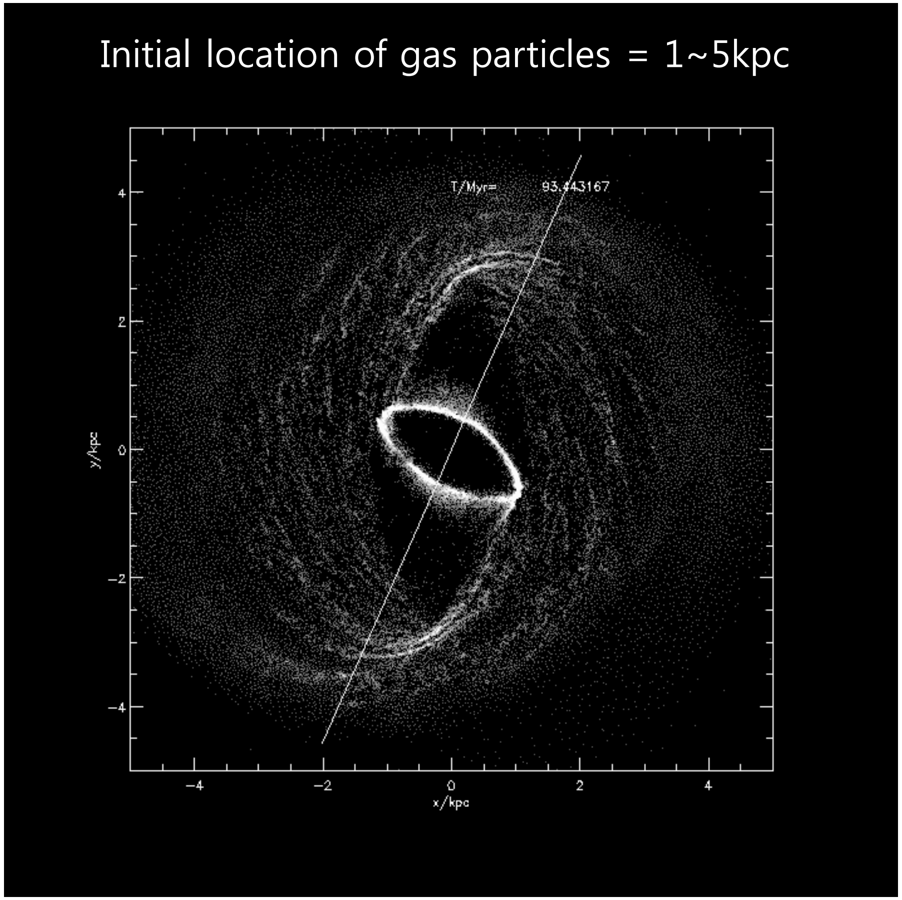
- calculation time is 40~50 times faster than live particles
- easy to get acceleration for any shapes of the galactic potential
- easy to change elongation, orientation, total mass, scale, etc.
- able to describe evolution of the realistic galactic potential as time

# Test Run – without spiral arms

In case of a snapshot without spiral arms + pattern speed of 45 km/s/kpc

$10^5$ gas particles following exponential density profile

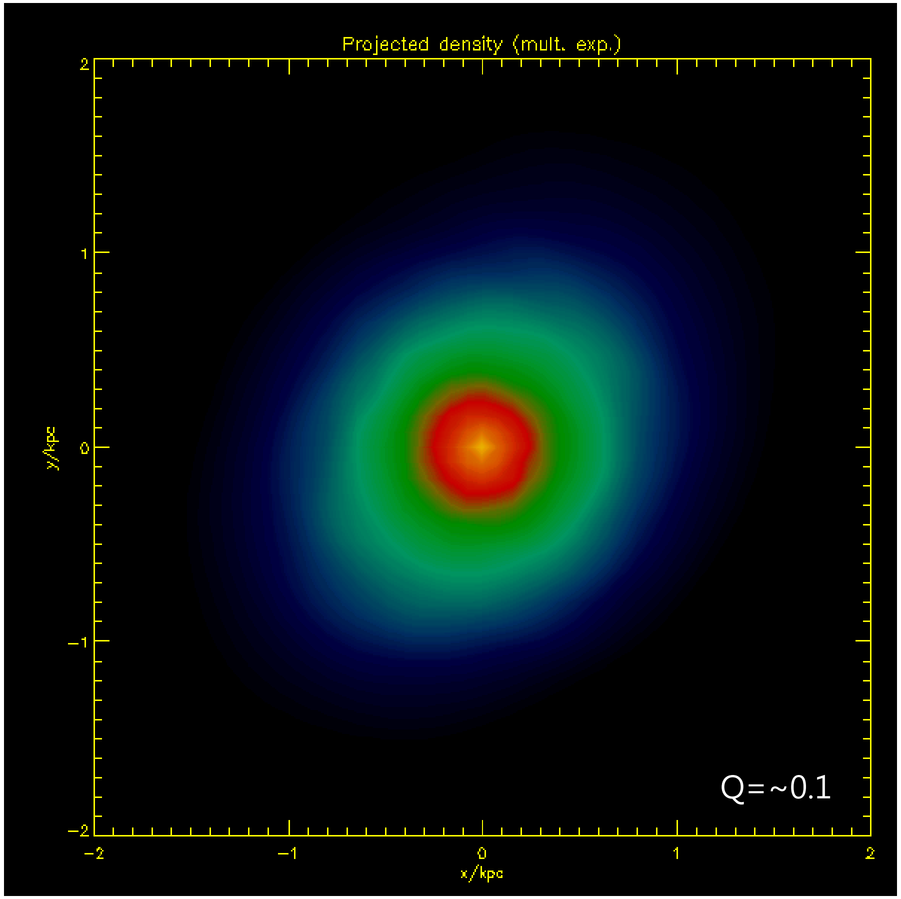


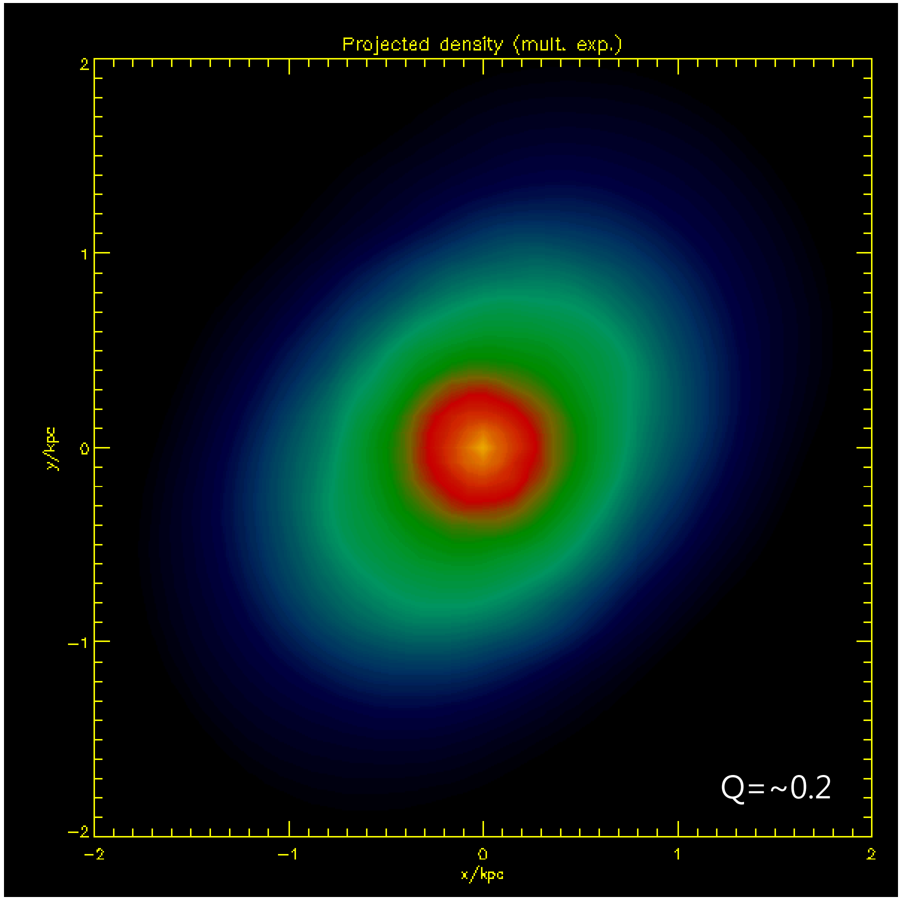



# Test Run – without spiral arms

Bar strength measured by Q parameter of $Q_b \equiv \left.\frac{F_T(r,\phi)}{F_R(r)}\right|_{\max}$

Higher Q value : increasing expansion coefficient of m=2 of the axi-symmetric bulge



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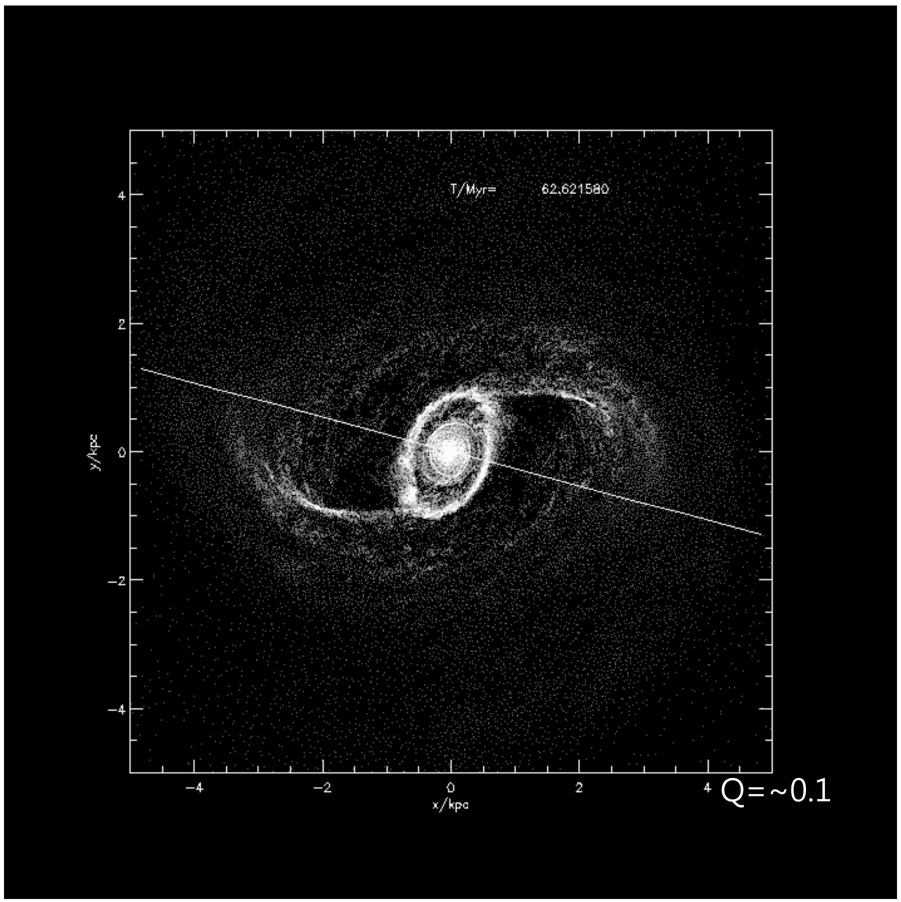
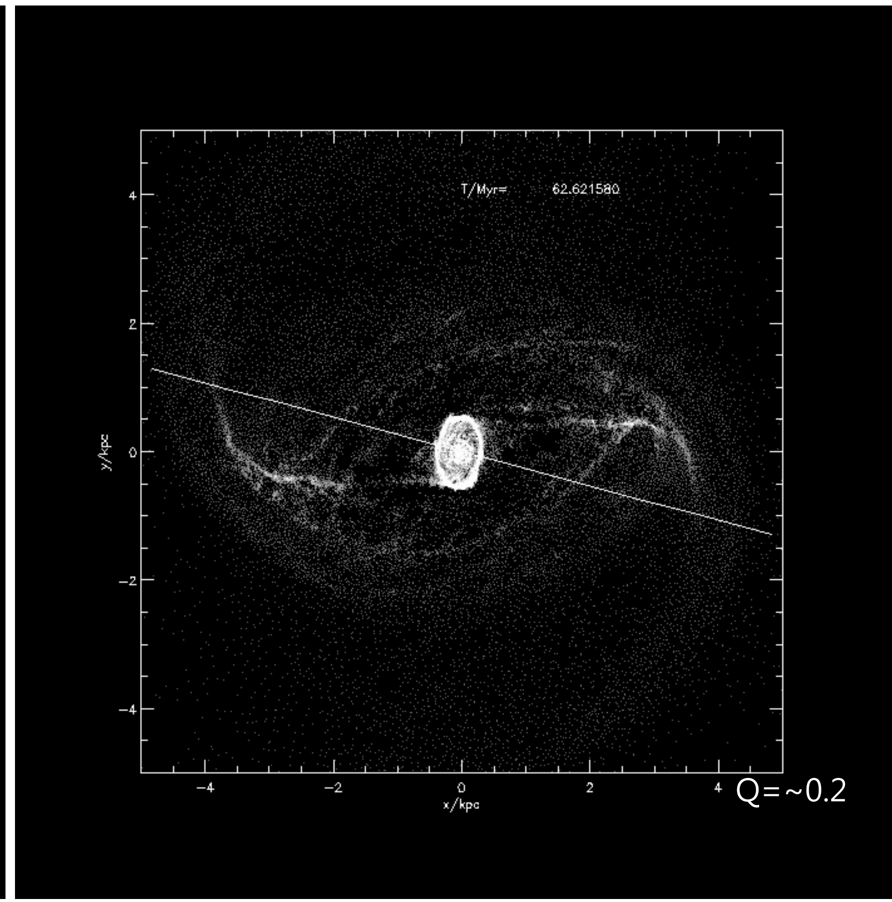
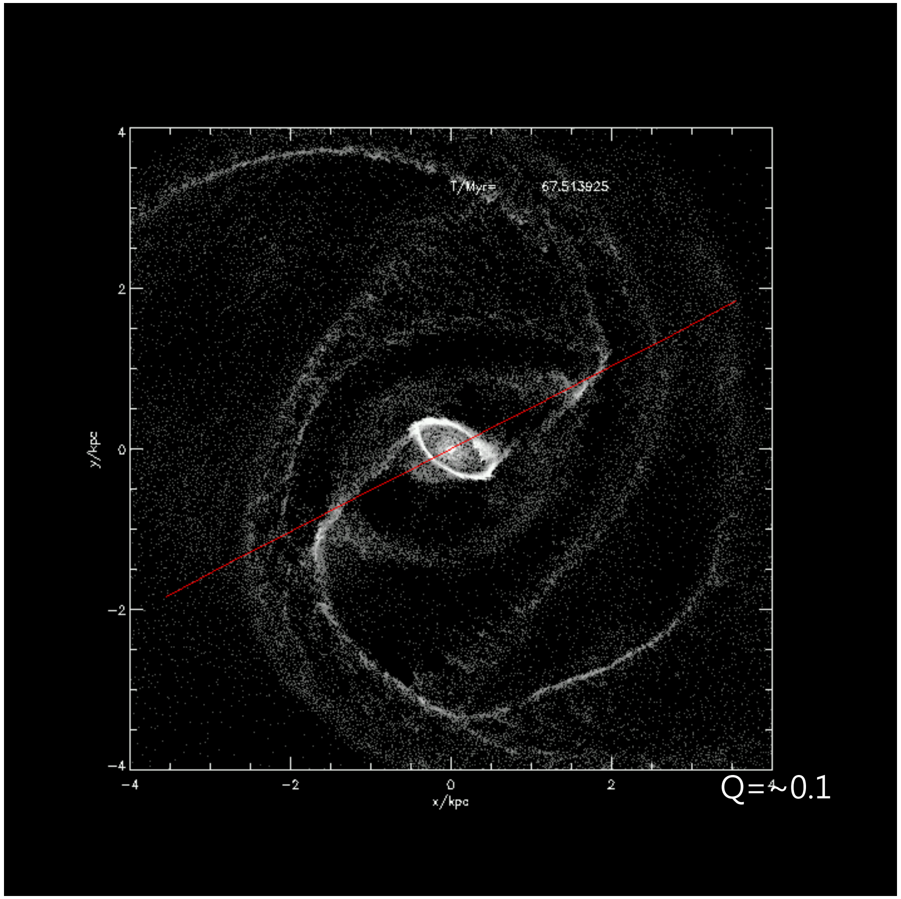
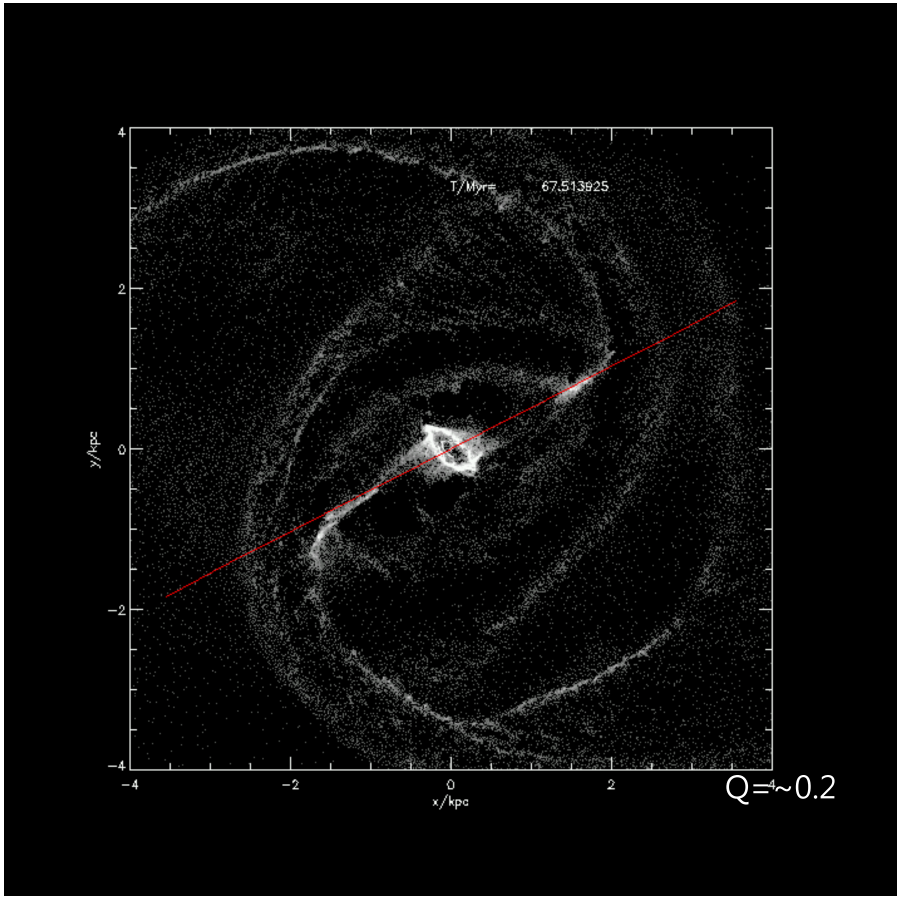


# Test Run – with spiral arms

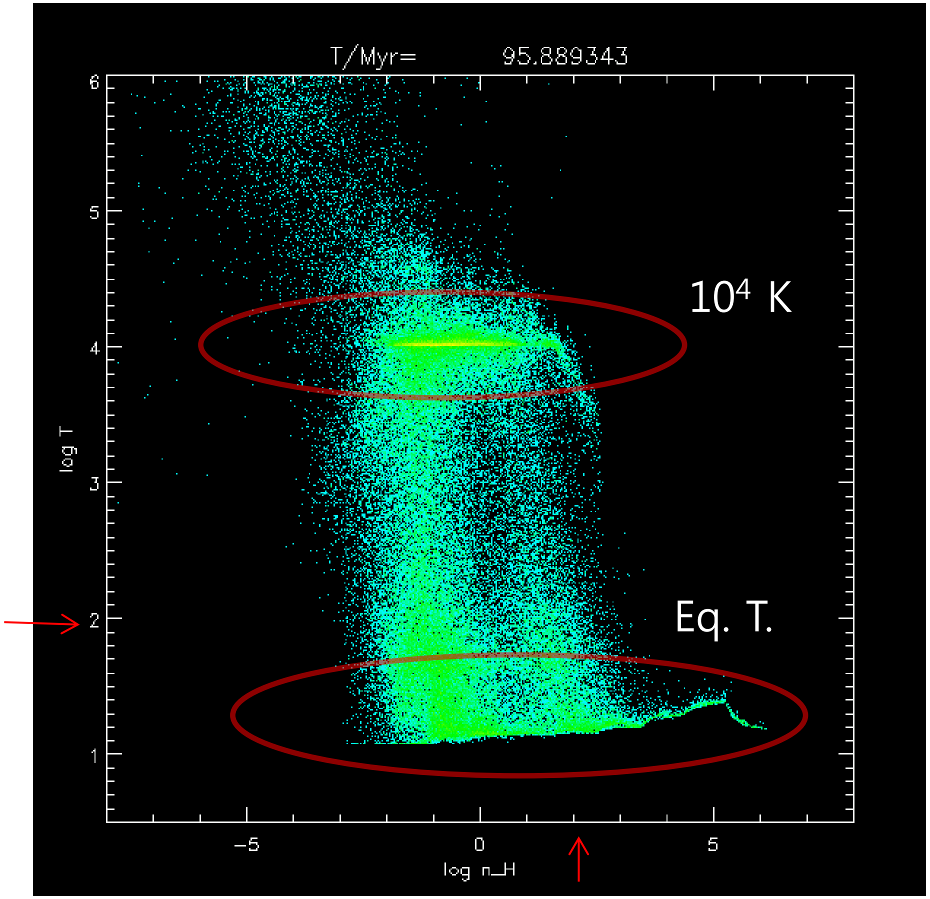


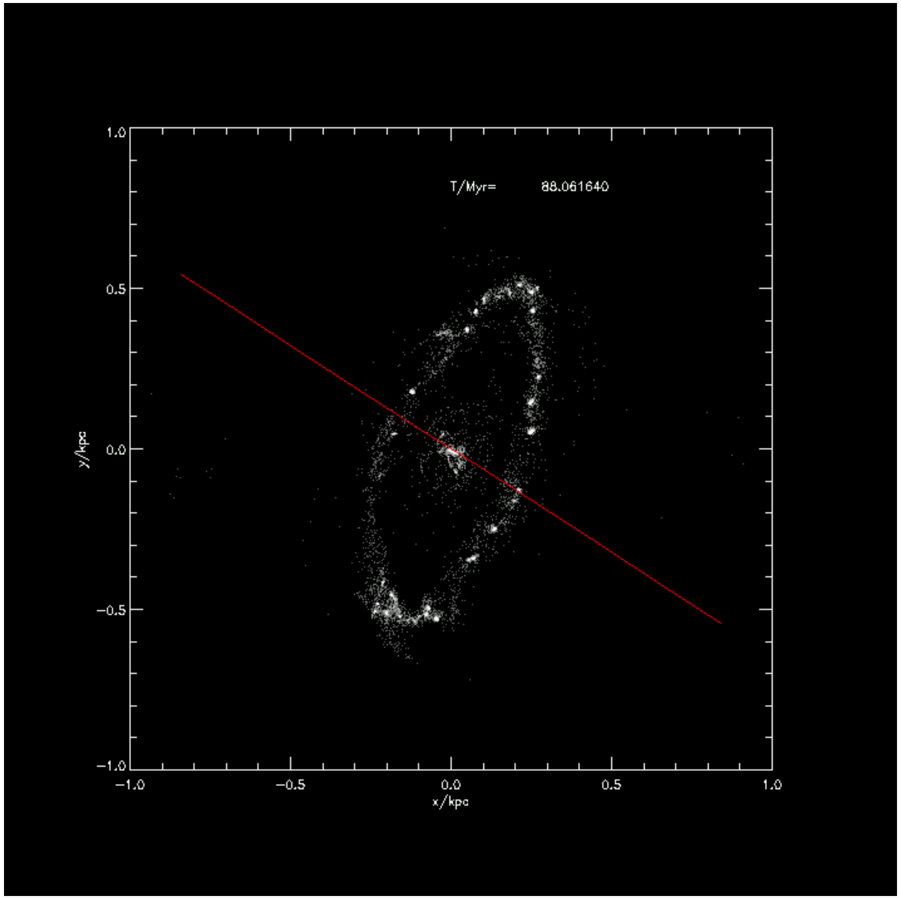



# Test Run – star formation

Most of gas particles are distributed near ~$10^4$ K or equilibrium temperature.

Stars are formed when $n_H > 100\ cm^{-3}$, $T < 100$ K $\nabla \cdot \mathbf{v} <$ 0.




$n_H$ = 100cm$^-$

# Future Work

Formation/evolution of nuclear ring

-> by different bar strength and/or existence of the spiral structures

-> origin of gas in the nuclear region (gas inflow from the galactic disk)

-> further inward migration of gas from CMZ to CND

-> star formation rate, star formation history in the nuclear region

-> nuclear bulge growth

With more realistic galactic potential

-> series of simulation snapshot, interpolating expansion coefficients

-> nested bar, fluctuating motions in the CMZ, twisted nuclear ring, etc.