# MHD turbulence dynamo

UNIST

**Kiwan Park and Dongsu Ryu**

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2. The annihilation of Magnetic field?

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→ **MHD turbulence dynamo & magnetic reconnection**

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## Questions are…

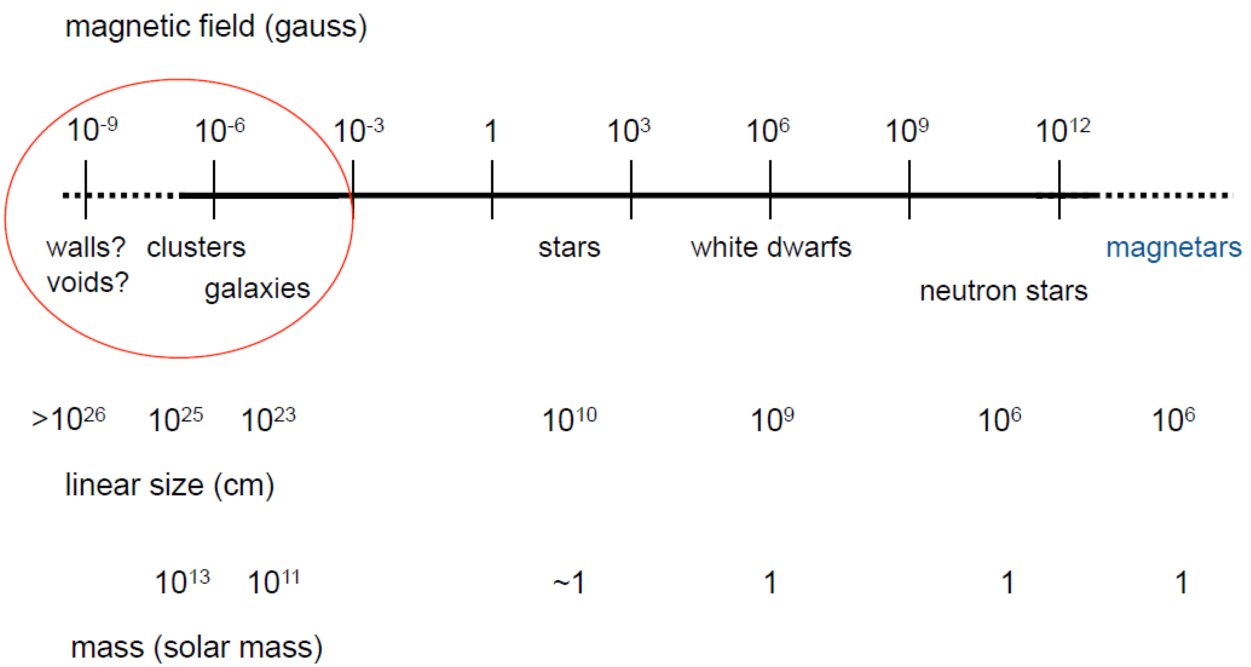
1. What are the origin and evolution of Magnetic field?
2. The annihilation of Magnetic field?

→ **MHD turbulence dynamo & magnetic reconnection**

- MHD turbulence converts
  mechanical energy (such as supernova explosion) to
  magnetic energy→ Galactic dynamo occurs.
- Magnetic reconnection converts magnetic energy to thermal or kinetic energy.

→ Heating source for the ISM and halo gas

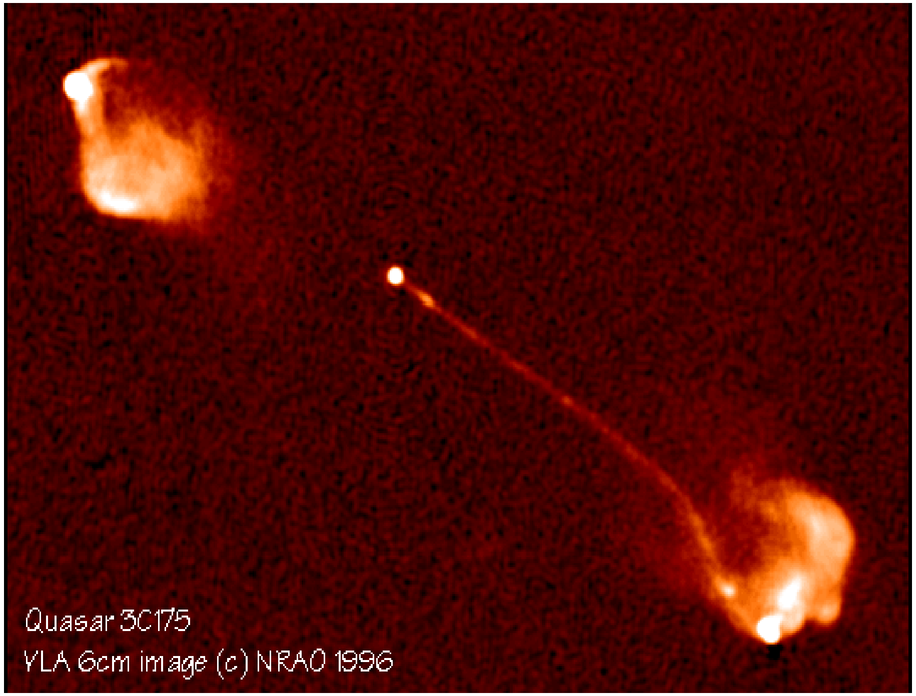
# Strength of magnetic field has wide range of magnitude $10^{-9}$G - $10^{12}$G

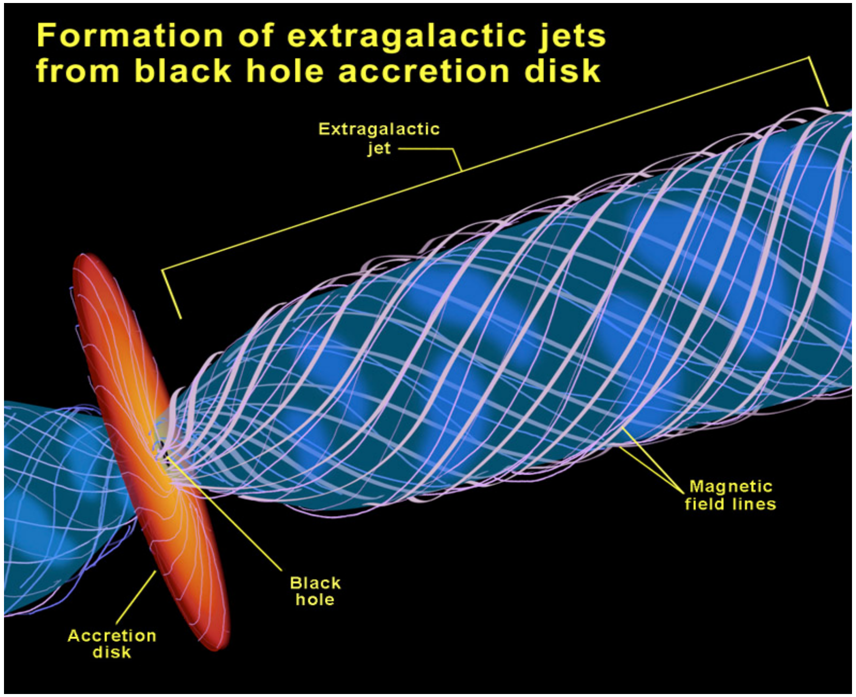



More examples
Spiral galaxy ~ 10 μG
Radio faint galaxy (M31, M33) & milky way galaxy ~ 5 μG
Gas rich spiral galaxy (M51) ~ 20-30 μG
Starburst galaxy (M82) ~ 50-100 μG

# Also magnetic field has huge range of scale

## Active Galactic Nucleus (AGN) Jet: 3C175



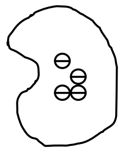

**The overall linear size: 212 kpc ($6.9\times10^5$ light year)**

**How does the magnetic field grow?**
**- Large scale dynamo & small scale dynamo**

# Process of MHD turbulence energy cascade

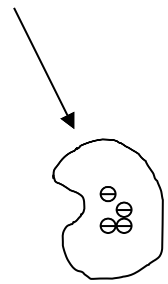
Suppose a bunch of ionized particles (plasmas)

**Kinetic Eddy**



# Process of MHD turbulence energy cascade

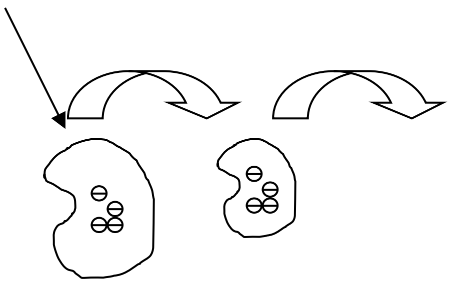
Mechnical forcing



**Kinetic Eddy**

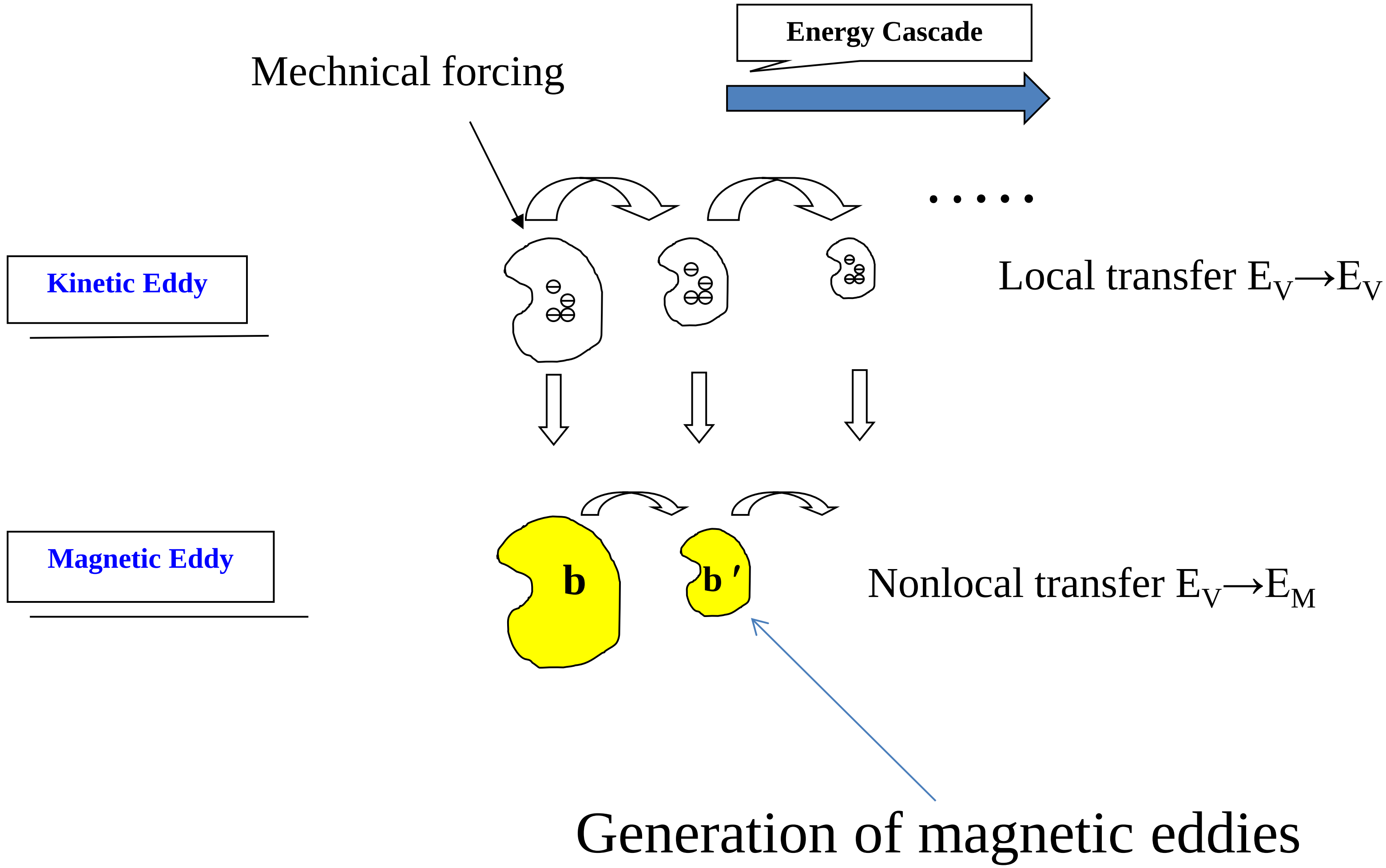
# MHD turbulence dynamo

Mechnical forcing



**Kinetic Eddy**

# MHD turbulence dynamo



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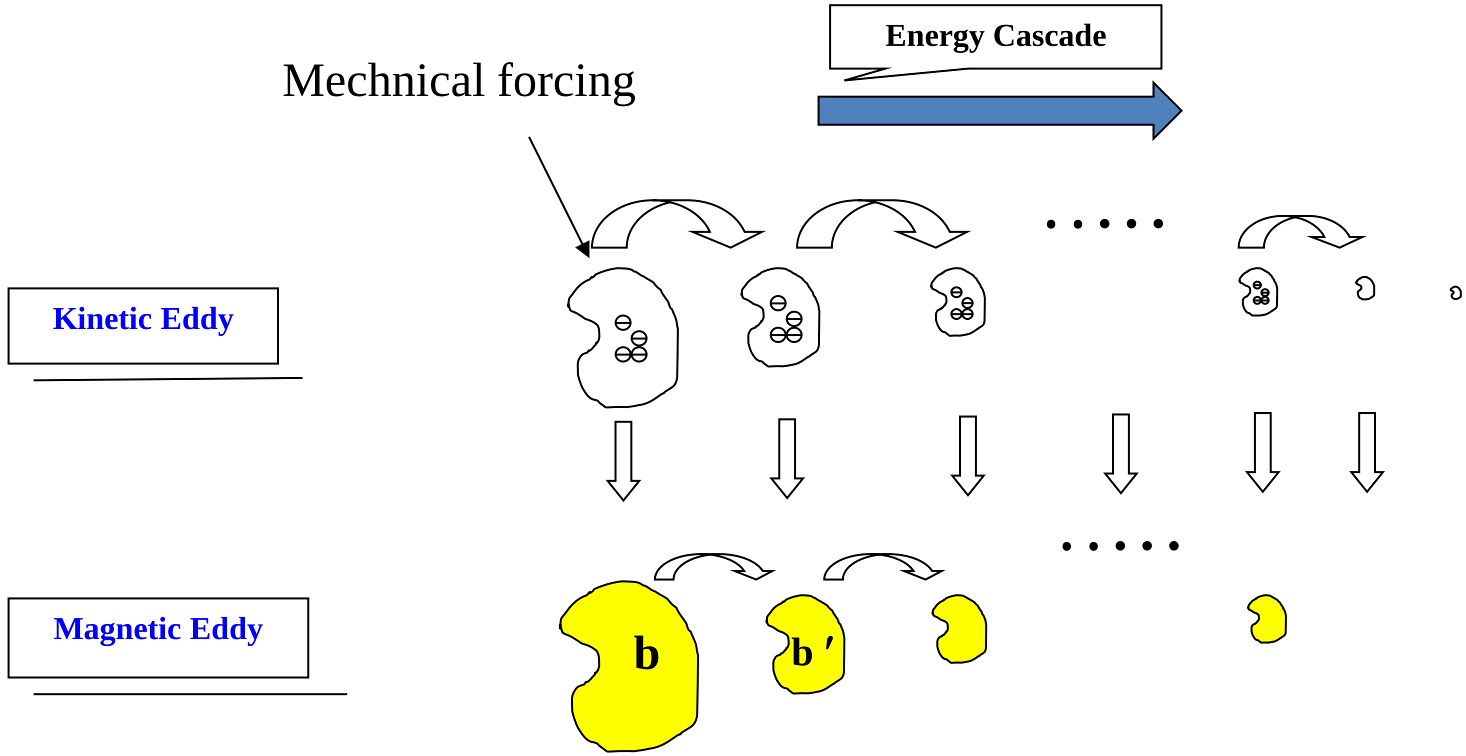

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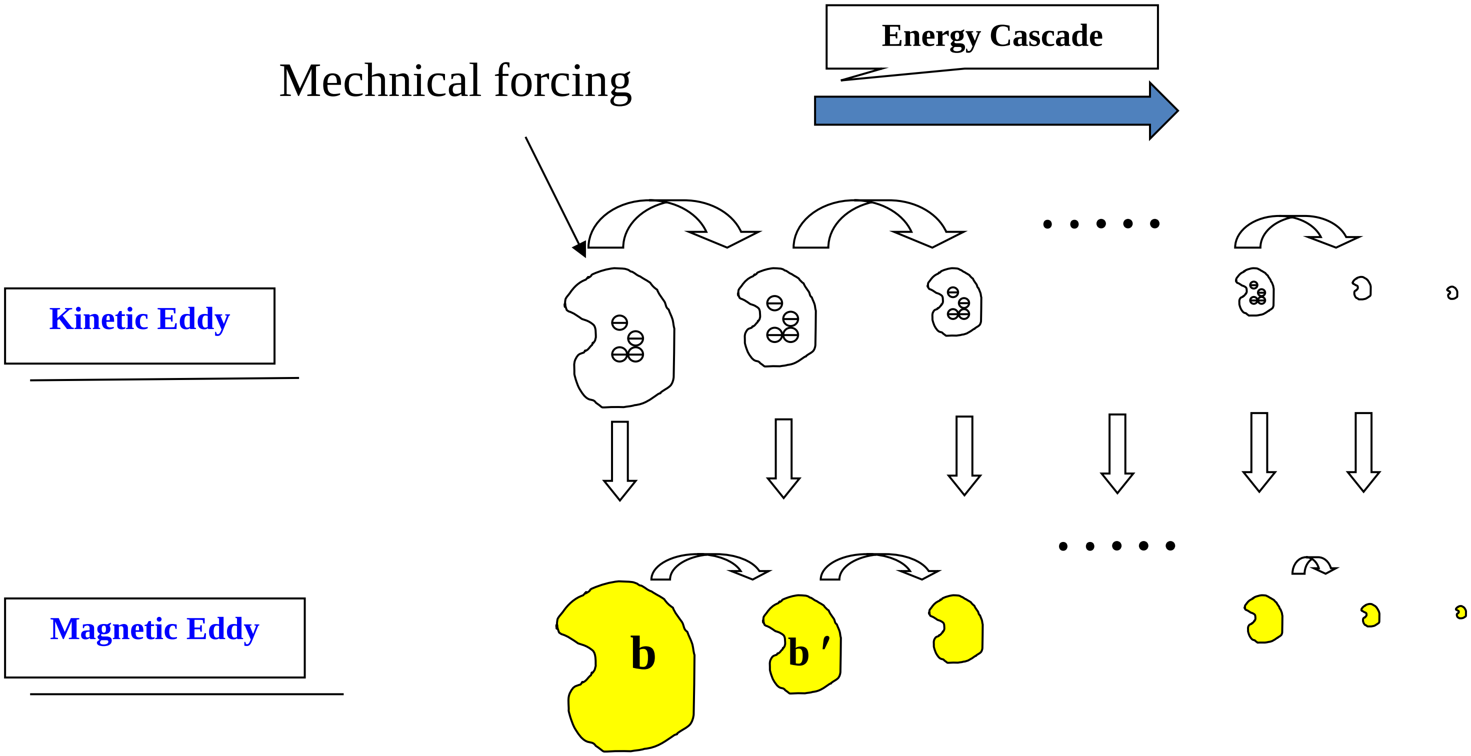
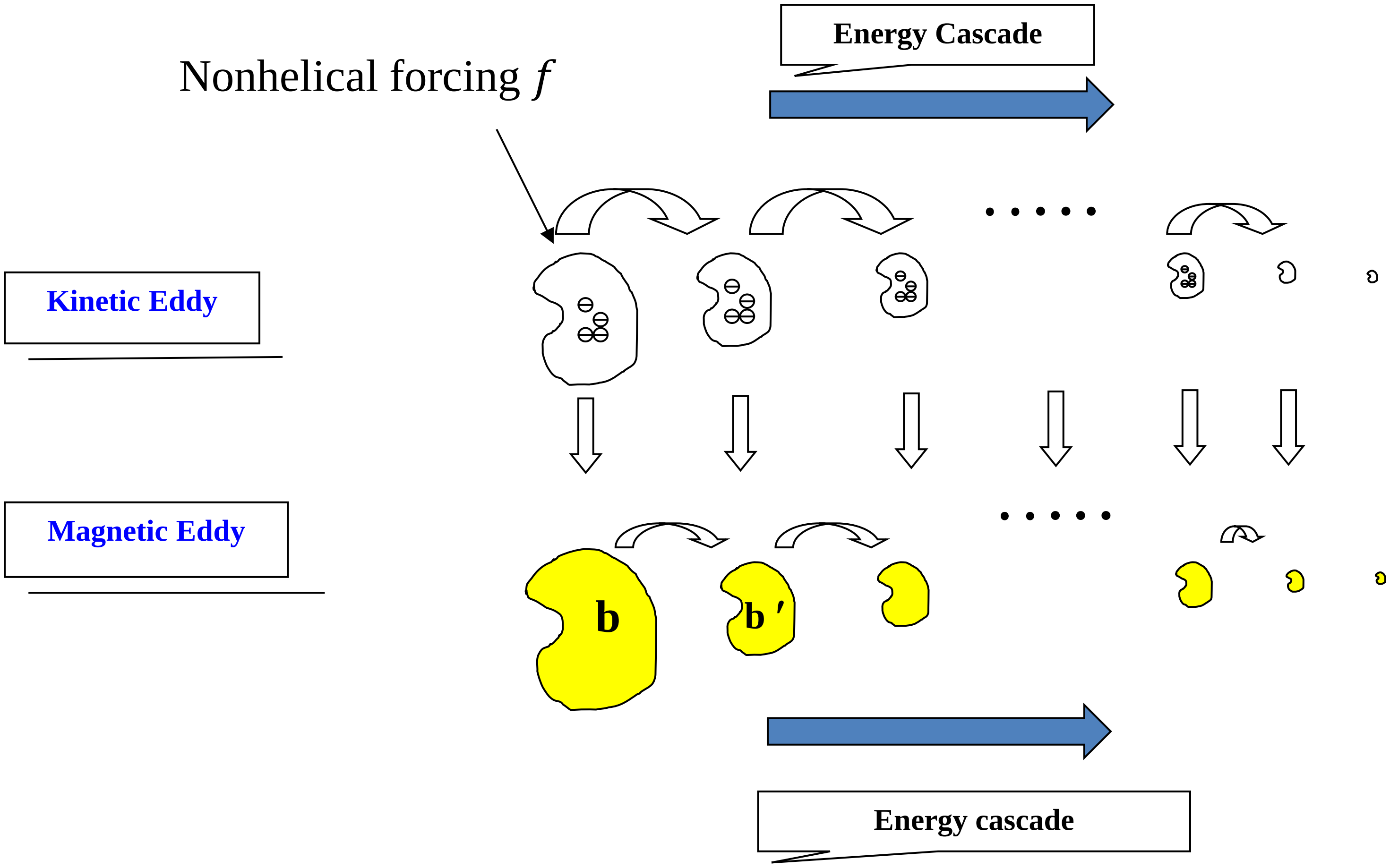
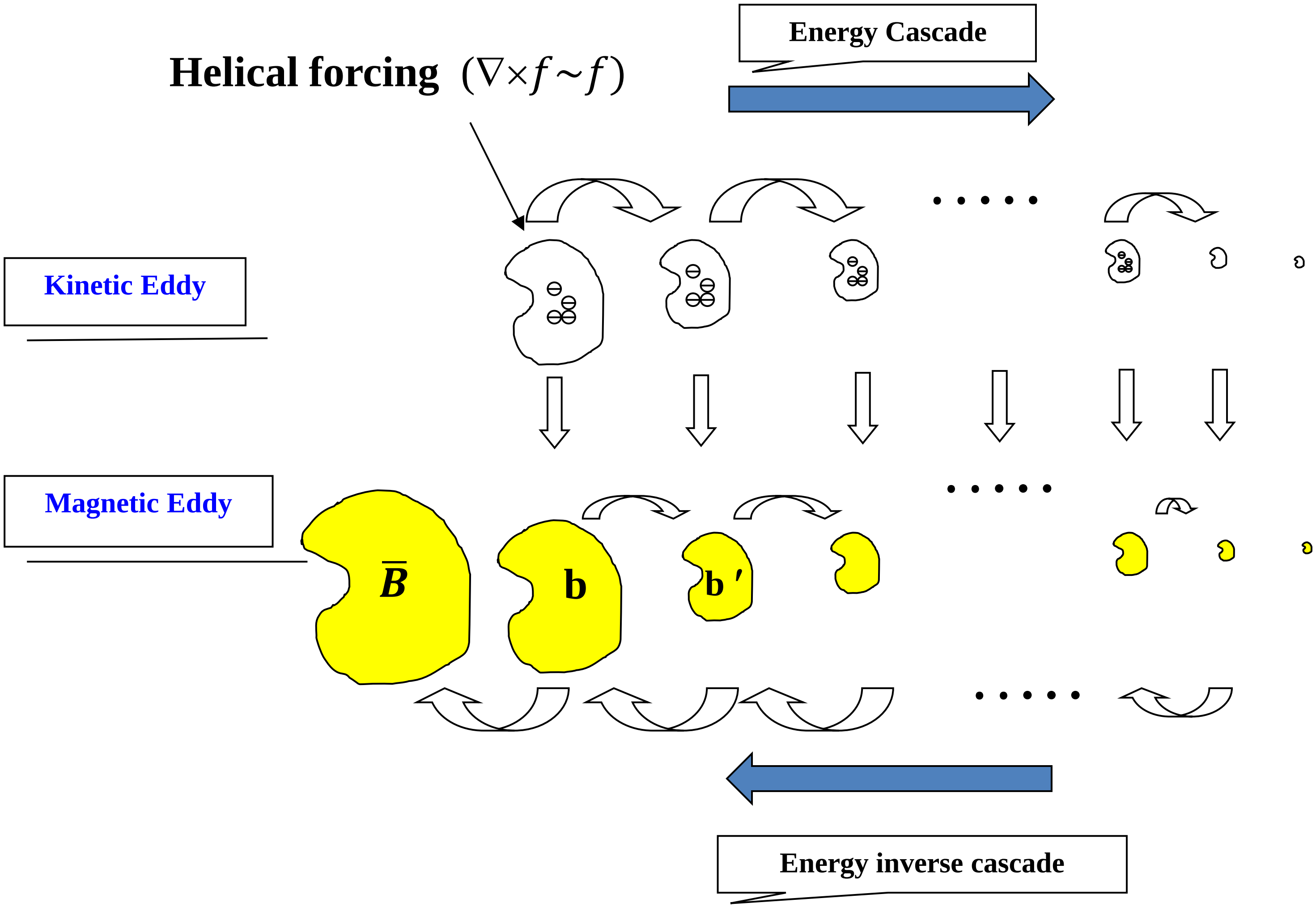


# MHD turbulence dynamo (small scale dynamo)




Large→small (small scale dynamo)

# MHD turbulence dynamo (large scale dynamo)




Small→Large (Large scale dynamo)

- **MHD equations**

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**1. Continuity Equation**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0$$

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**2. Momentum equation**

$$\rho \frac{DV}{Dt} = -\nabla P + \frac{1}{c} J \times B + \nu \nabla^2 V \qquad \left( \frac{D}{Dt} \equiv \frac{\partial \rho}{\partial t} + V \cdot \nabla \right)$$

- **MHD equations**

### 1. Continuity Equation

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### 3. Magnetic induction Equation

$$\frac{\partial B}{\partial t} = \nabla \times \underbrace{\langle V \times B \rangle}_{\text{EMF}} + \eta \nabla^2 B$$

- **For the intuitive understanding, we need more simplified magnetic induction equation.**

**→Mean field (two scale) model is useful**

$\boldsymbol{V} \rightarrow \bar{\boldsymbol{V}}$ (large) + **v** (fluctuation)

$\boldsymbol{B} \rightarrow \bar{\boldsymbol{B}}$ (large) + **b** (fluctuation)

- **Large scale dynamo**

**- inverse cascade of $\boldsymbol{E_M}$ (small → large scale)**

$\bar{\boldsymbol{B}}$ (large) + **b** (fluctuation)

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

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$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

$\bar{\boldsymbol{V}}$ (large) + **v** (fluctuation)

$$\rightarrow \frac{\partial \overline{B}}{\partial t} = \underbrace{\nabla \times \langle v \times b \rangle}_{\text{Source (small)}} + \underbrace{(\eta + \beta) \nabla^2 \overline{B}}_{\text{Diss.}}$$

Elctromotive force (EMF) $\langle v \times b \rangle$

# • Large scale dynamo

## - inverse cascade of $E_M$ (small → large scale)

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When field is helical ($\nabla \times b \sim b$),
EMF can be represented by $\alpha$, $\beta$

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$$\frac{dv}{dt} \sim \langle \bar{J} \times b + j \times \bar{B} \rangle$$

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$$\frac{dv}{dt} \sim \langle \overline{J} \times b + j \times \overline{B} \rangle \qquad \frac{db}{dt} \sim \nabla \times \langle v \times \overline{B} \rangle$$

# • Large scale dynamo

## - inverse cascade of $E_M$ (small → large scale)

$\bar{\boldsymbol{B}}$ (large) + **b** (fluctuation)

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times \langle \mathrm{V} \times \mathrm{B} \rangle + \eta \nabla^2 \mathrm{B}$$

$\bar{\boldsymbol{V}}$ (large) + **v** (fluctuation)

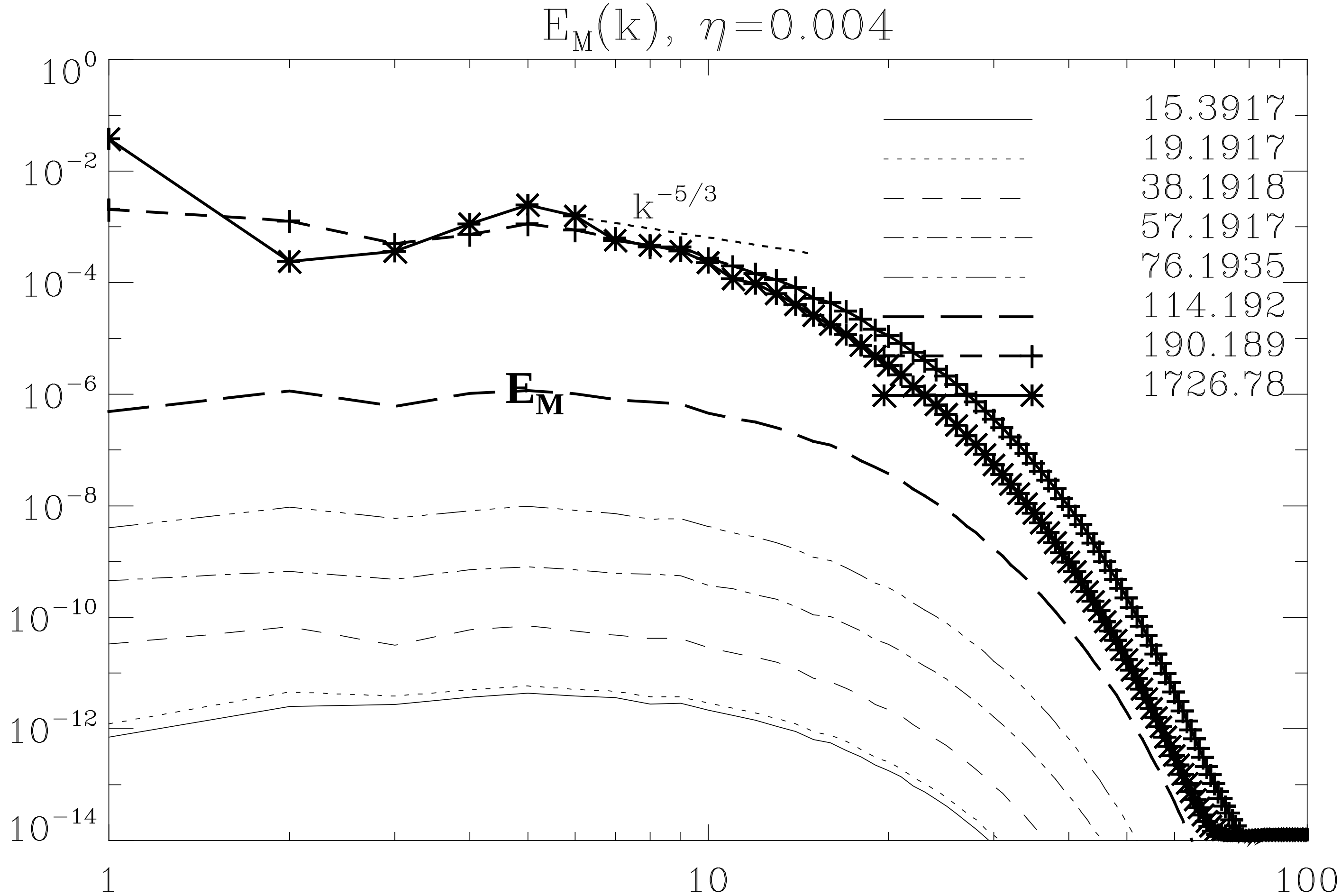
$$\rightarrow \frac{\partial \overline{\mathrm{B}}}{\partial t} = \underbrace{\nabla \times \langle \mathrm{v} \times \mathrm{b} \rangle}_{\text{Source(small)}} + \underbrace{(\eta + \beta) \nabla^2 \overline{\mathrm{B}}}_{\text{Diss.}}$$

$$\rightarrow \frac{\partial \overline{\mathrm{B}}}{\partial t} = \underbrace{\nabla \times \alpha \overline{\mathrm{B}}}_{\text{Source}} + \underbrace{(\beta + \eta) \nabla^2 \overline{\mathrm{B}}}_{\text{Dissipation}}$$

**Helical forcing**

**($\nabla \times \boldsymbol{v} \sim \boldsymbol{v}$ *or***

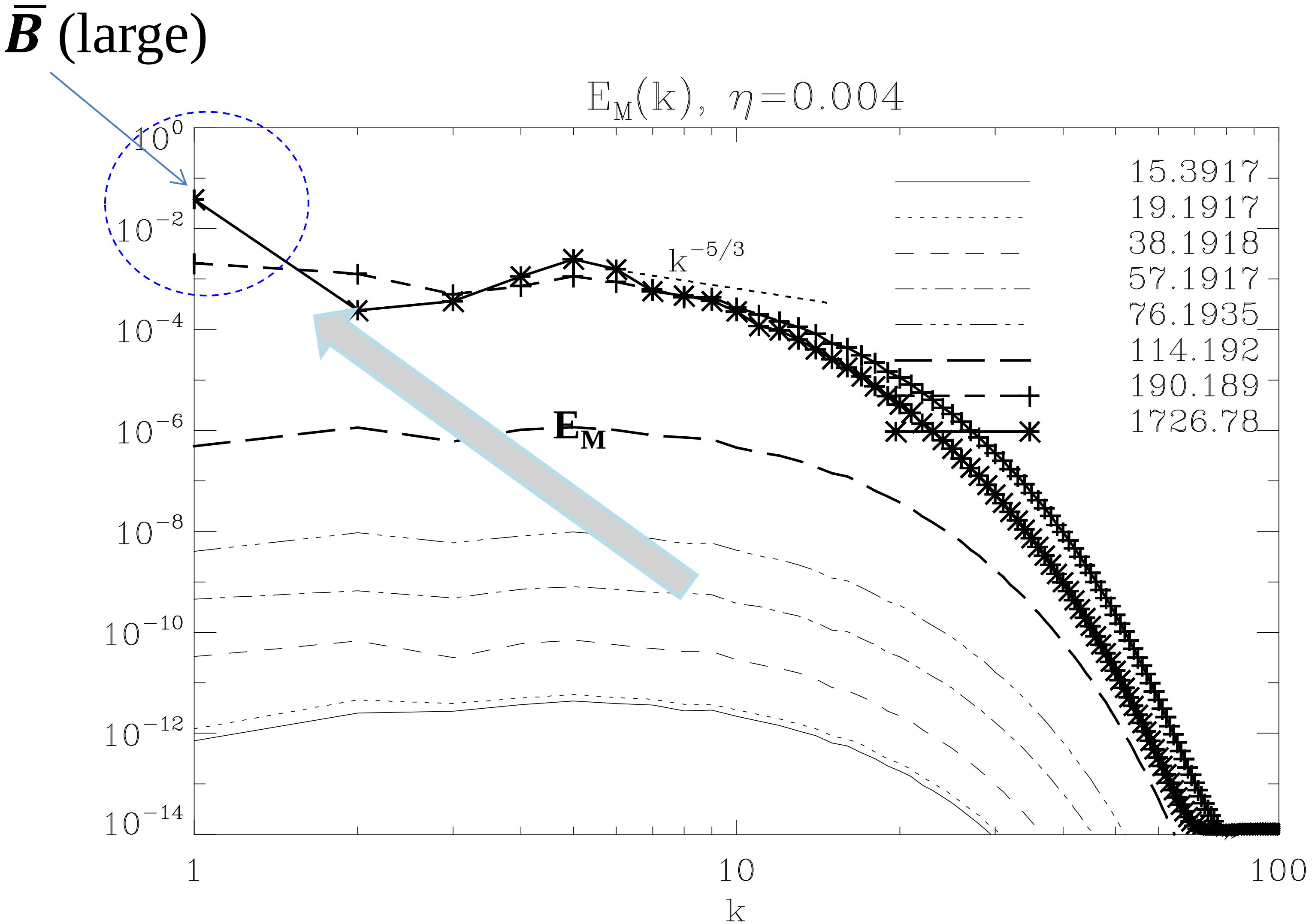
# $E_M$ for Large scale dynamo (helical forcing)




**Wave number** k

**Large scale** **Small scale**

# $E_M$ for Large scale dynamo (helical forcing)



**However, in fact mechanical force like**

**$E_V$ of supernova has 5-15% helicity**

**→ mostly nonhelical, forward cascade of $E_M$**

**may be dominant → small scale dynamo**

- **Small scale dynamo (nonhelical field)**

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

$$= B \cdot \nabla V - V \cdot \nabla B + \eta \nabla^2 B$$

- **Small scale dynamo (nonhelical field)**

$$\frac{\partial B}{\partial t} = \nabla \times \left\langle V \times B \right\rangle + \eta \nabla^2 B$$

$$= B \cdot \nabla V - V \cdot \nabla B + \eta \nabla^2 B$$

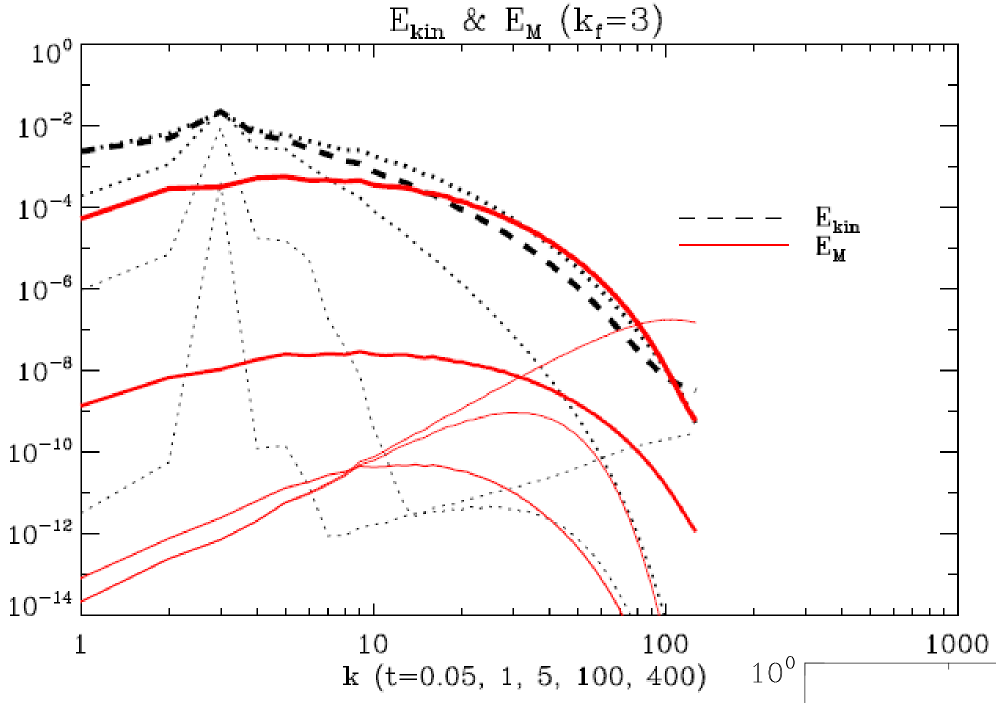
$\rightarrow$ **Kazantsev equation…**

**assumes** $\langle vv \rangle = \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \kappa$

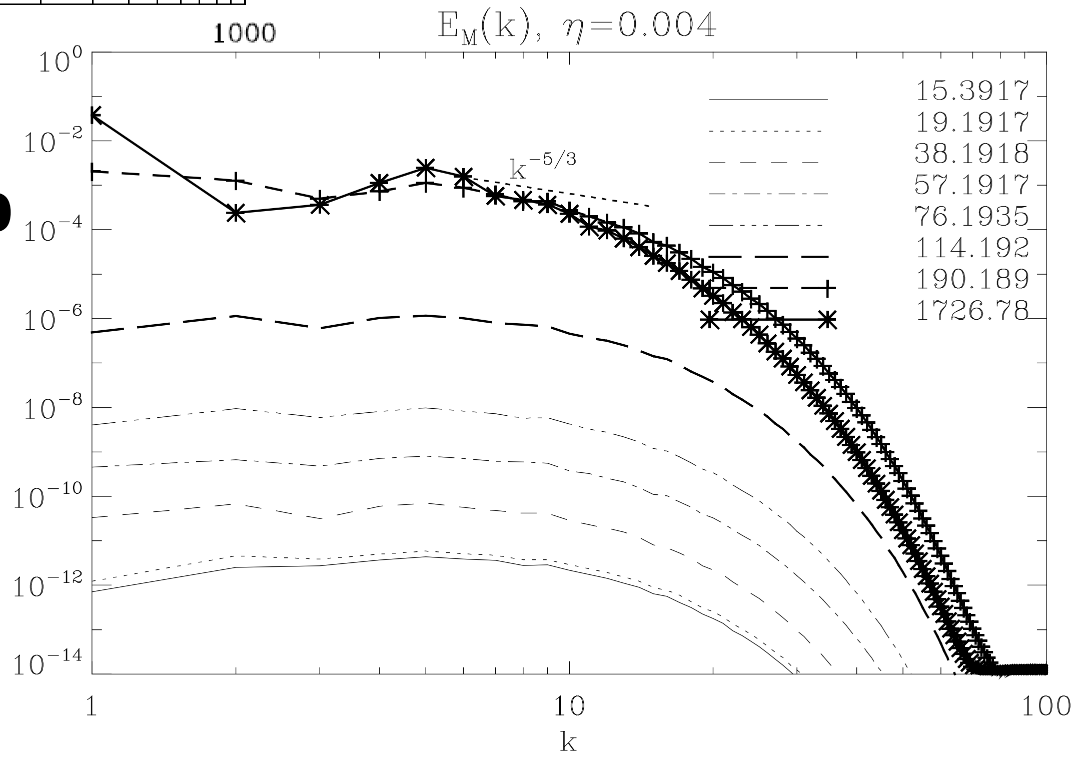
- **Comparison of large scale & small scale dynamo**

  **- Magnetic energy spectrum** $E_M$

# Small scale dynamo




# Large scale dynamo



- **How about the small scale dynamo in high prandtl number?**

**(suitable for warm & partially ionized galaxies ~ $10^{14}$ )**

$$\rho \frac{DV}{Dt} = -\nabla P + \frac{1}{c} J \times B + \nu \nabla^2 V$$

$$Pr_M \equiv \frac{\nu}{\eta} \gg 1$$

$$\frac{\partial B}{\partial t} = \nabla \times \underbrace{\langle V \times B \rangle}_{\text{EMF}} + \eta \nabla^2 B$$

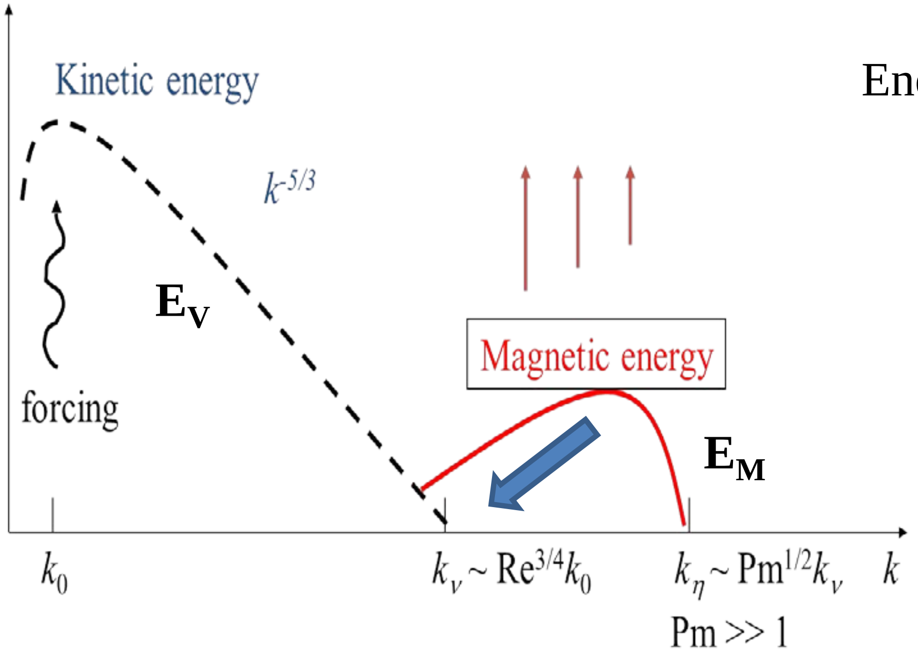
- **Small scale dynamo in high prandtl number**

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

$$\sim B \cdot \nabla V - V \cdot \nabla B$$

**$\eta \sim 0$**

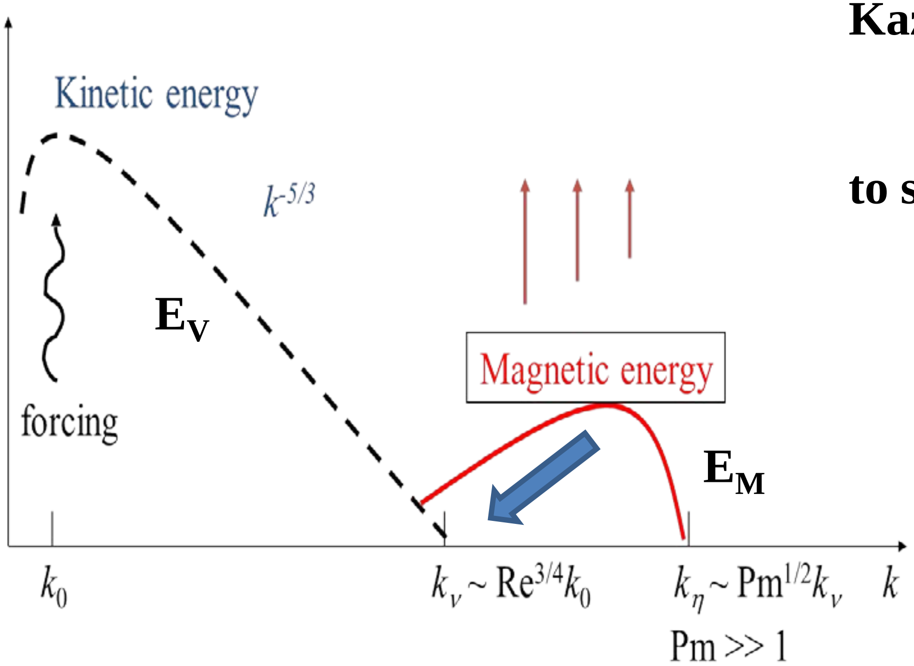
- **$E_M$ & $E_V$ in high prandtl number**




Energy spectrum

**Schekochihin et al. 2004**

# • $E_M$ & $E_V$ in high prandtl number




**Schekochihin et al. 2004**

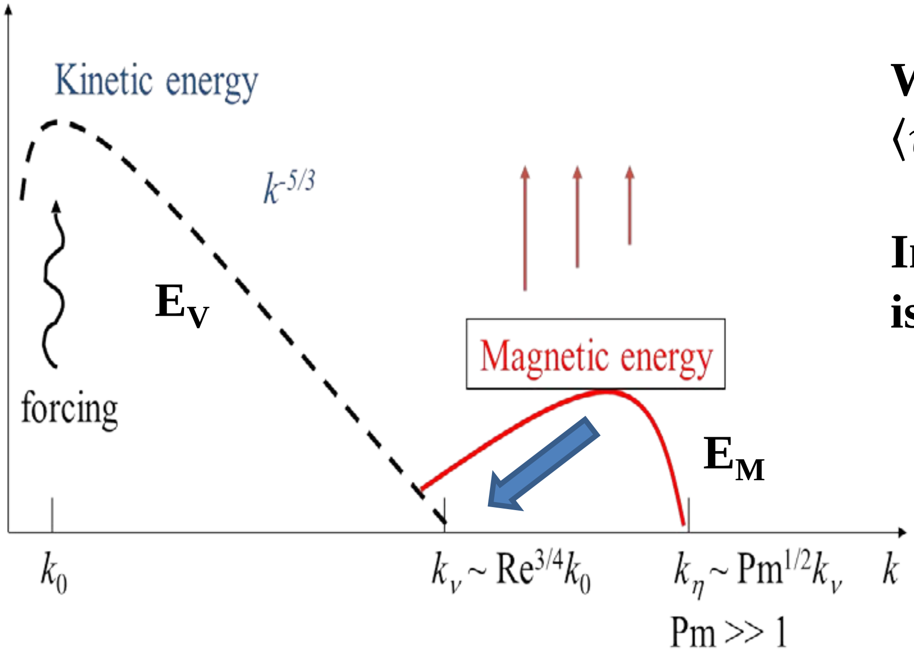
**Kazantsev's model assumes**

$$\langle vv \rangle = (\delta_{ij} - \frac{k_i k_j}{k^2})\kappa$$

**to solve (iterative method)**

$$\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}} = \boldsymbol{B} \cdot \nabla \boldsymbol{v} - \boldsymbol{v} \cdot \nabla \boldsymbol{B} + \eta \nabla^2 \boldsymbol{B}$$

- $E_M$ & $E_V$ in high prandtl number

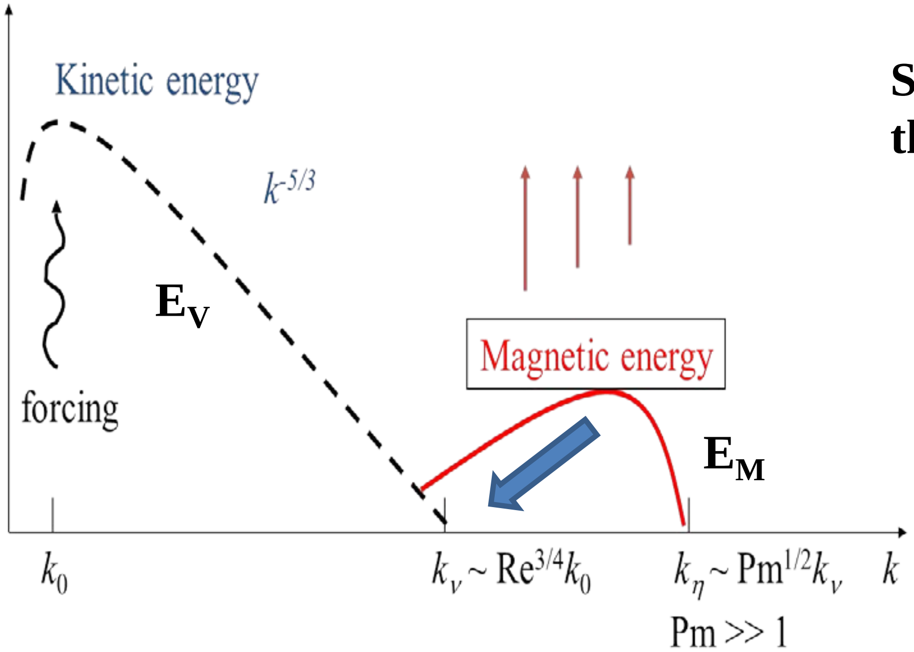



**Schekochihin et al. 2004**

**We don't know when dynamo begins, before $\langle vv \rangle$ or later.**

**In addition, the high $Pr_M$ means viscosity $\nu$ is large. Difficult to form $\langle vv \rangle$**

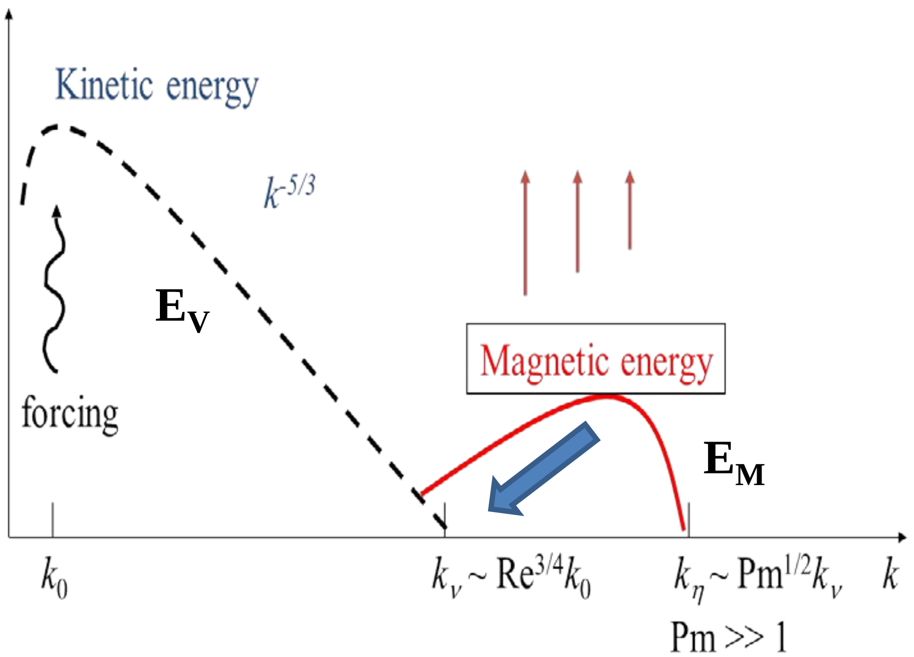
- **$E_M$ & $E_V$ in high prandtl number**




**Schekochihin et al. 2004**

**So, without pre-existing $\langle vv \rangle$, the spectrum is**

# • $E_M$ & $E_V$ in high prandtl number

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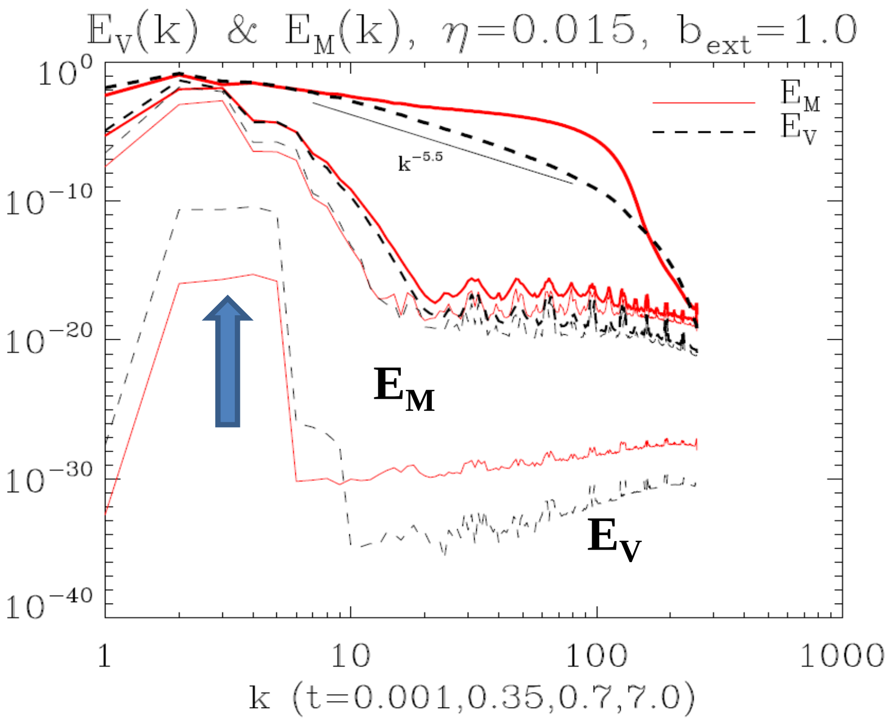
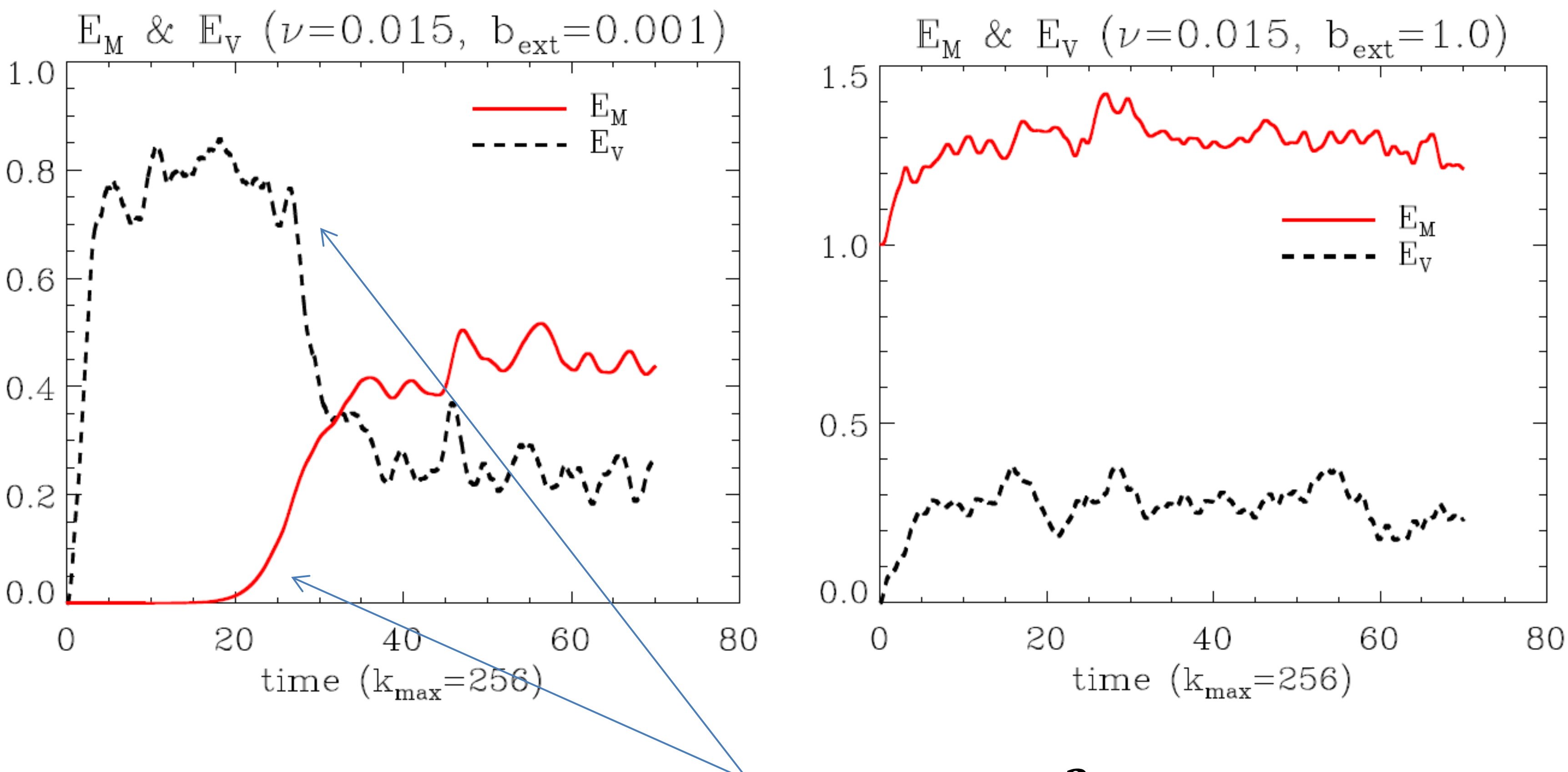



**Kiwan Park & Donsu Ryu 2014**

**Influence of strong and weak background magnetic field**
**Background magnetic field $B_{ext}$ affects the evolution of magnetic field profile.**

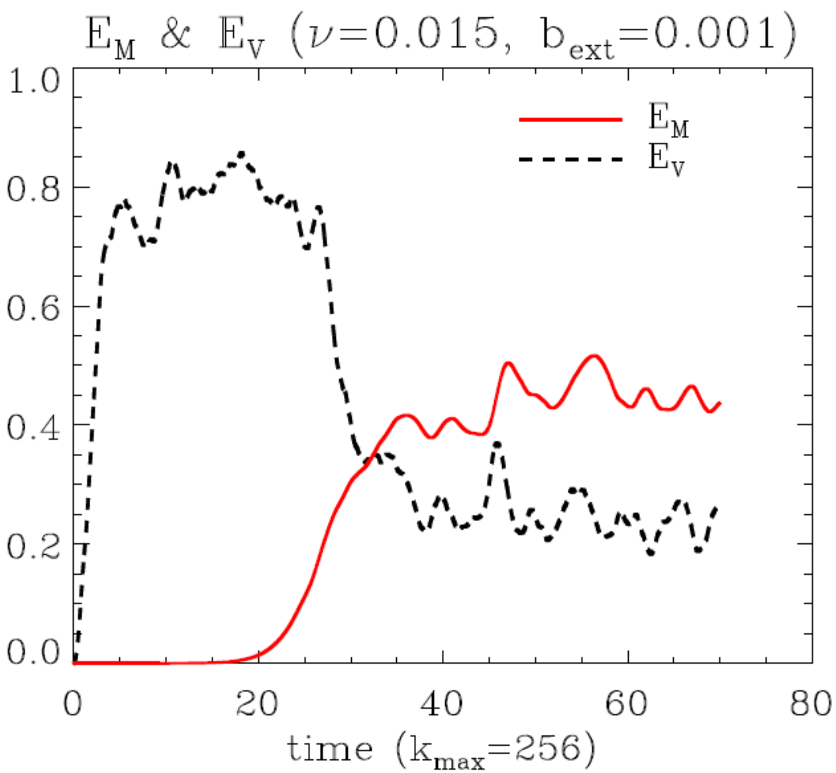
# Influence of background field on the evolution of fields

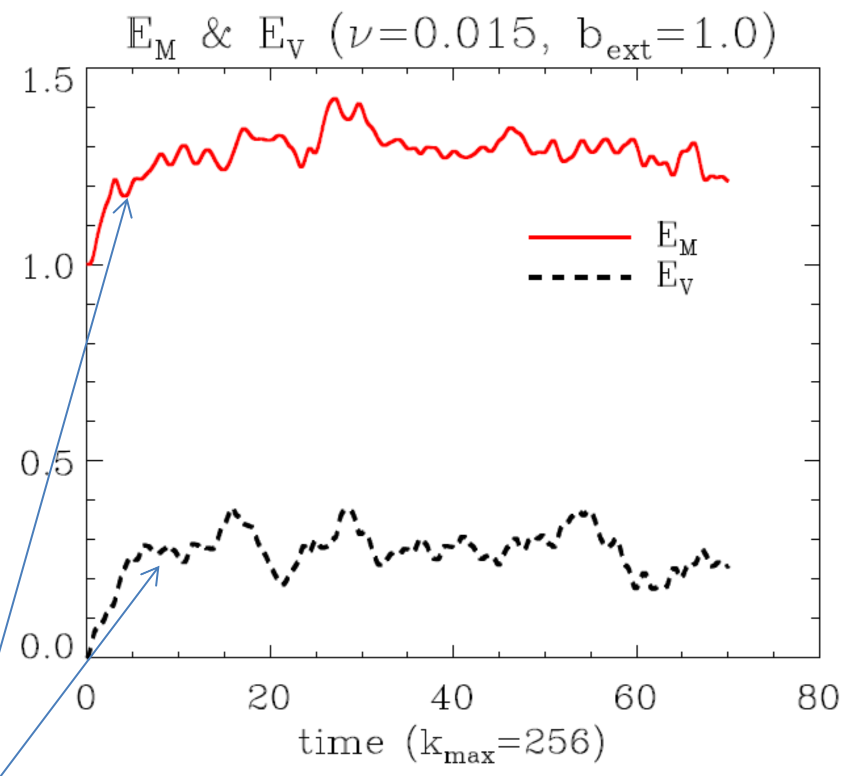



**The effect of magnetic pressure ($-\nabla B^2/2$) and tension ($\boldsymbol{B}\cdot\nabla\boldsymbol{B}$) is conspicuous with weak $\mathrm{B}_{ext}$(0.001).**

$$\rho\frac{DV}{Dt} = -\nabla\left(P + \frac{B^2}{2}\right) + B\cdot\nabla B + \nu\nabla^2 V.$$

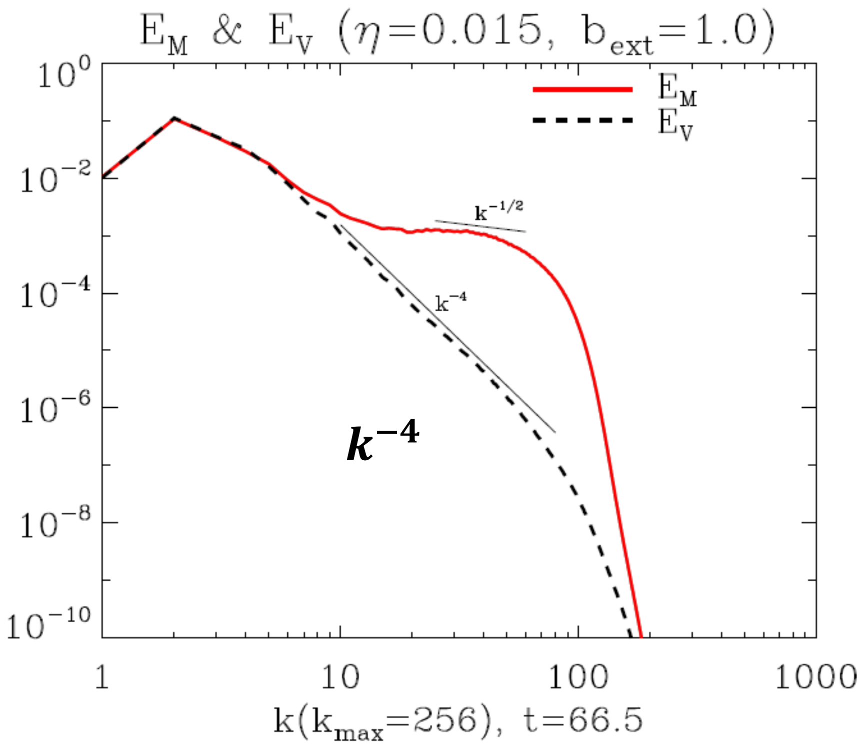
# Influence of background field on the evolution of fields

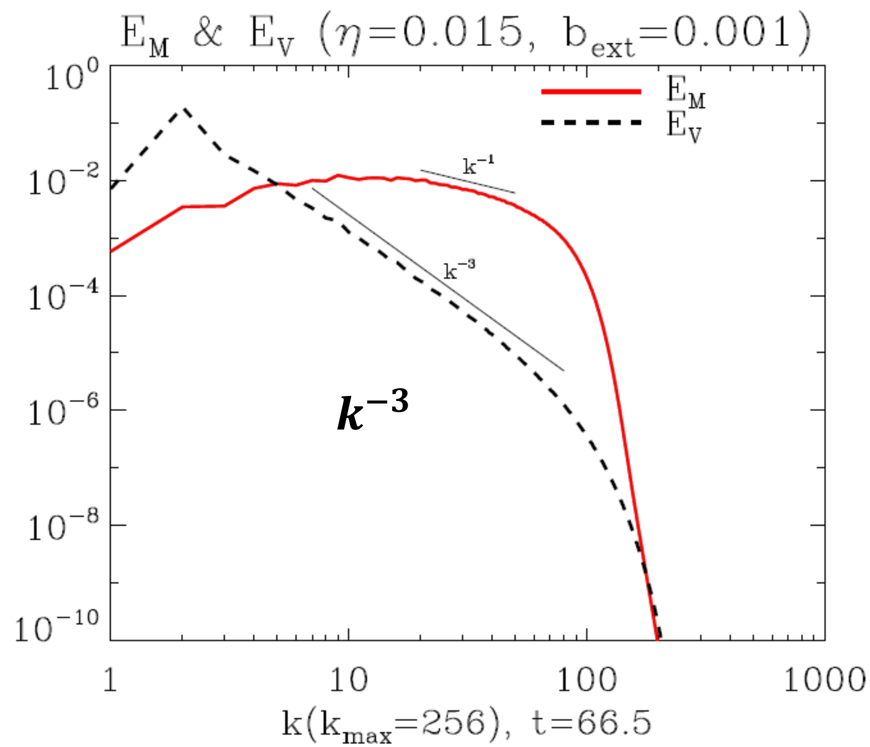



**But with strong B$_{ext}$(1.0), the constraint of plasma motion is not clearly shown.**

# Scaling law of $E_M$ and $E_V$ is influenced by $B_{ext}$




**1. Kolmogorov's assumption and the scaling law ($k^{-5/3}$) are not correct in small scale MHD dynamo with high $Pr_M$.**

**2. Scaling factor depends on $B_{ext}$**

- $E_V$($\boldsymbol{k}^{-4}$) and $E_M$($\boldsymbol{k}^{-1/2}$) with strong $\mathbf{B}_{ext}$(1.0)
- $E_V$($\boldsymbol{k}^{-3}$) and $E_M$($\boldsymbol{k}^{-1}$) with weak $\mathbf{B}_{ext}$(0.001)

# How to solve?

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

$\eta$ **~ 0**

$$\sim B \cdot \nabla V - V \cdot \nabla B$$

Dimensional analysis?

$$\frac{\partial B}{\partial t} = \nabla \times \langle V \times B \rangle + \eta \nabla^2 B$$

$$\sim B \cdot \nabla V - V \cdot \nabla B$$

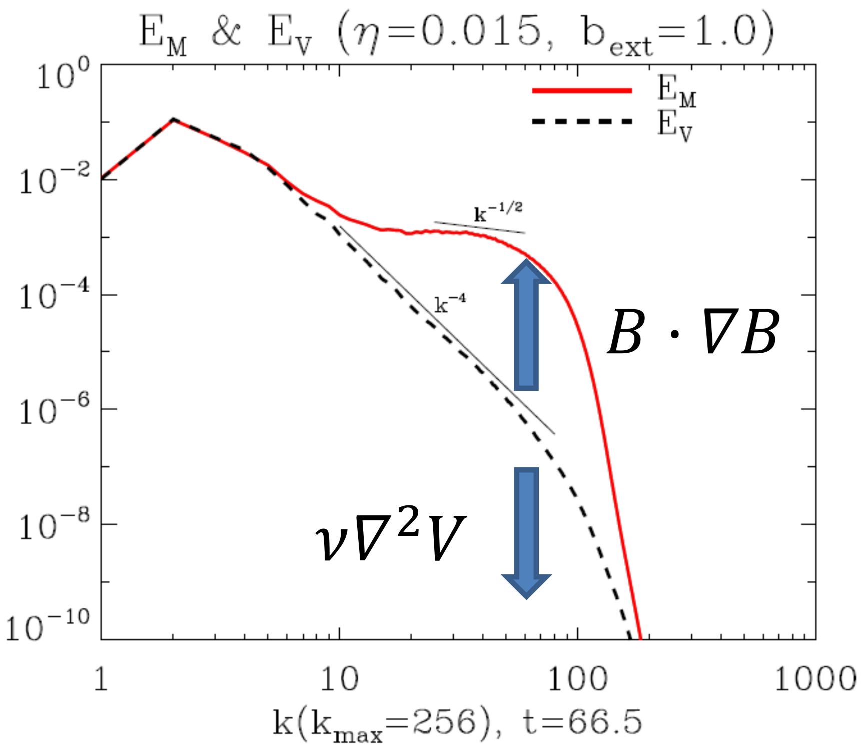
$$\rightarrow B\,k\,V - V\,k\,B$$

Dimensionally self consistent.

Instead…

# 1. Balance relation → Magnetic tension ~ dissipation

$$B \cdot \nabla B \sim \nu \nabla^2 V$$



1. **Magnetic tension ~ dissipation**

$$B \cdot \nabla B \sim \nu \nabla^2 V$$

2. **Magnetic energy transfer rate in stationary state**

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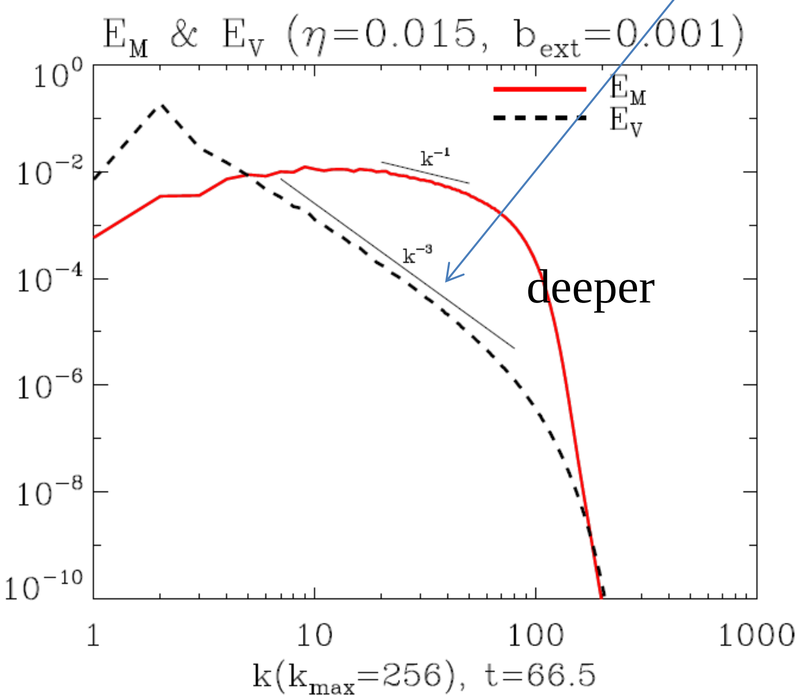
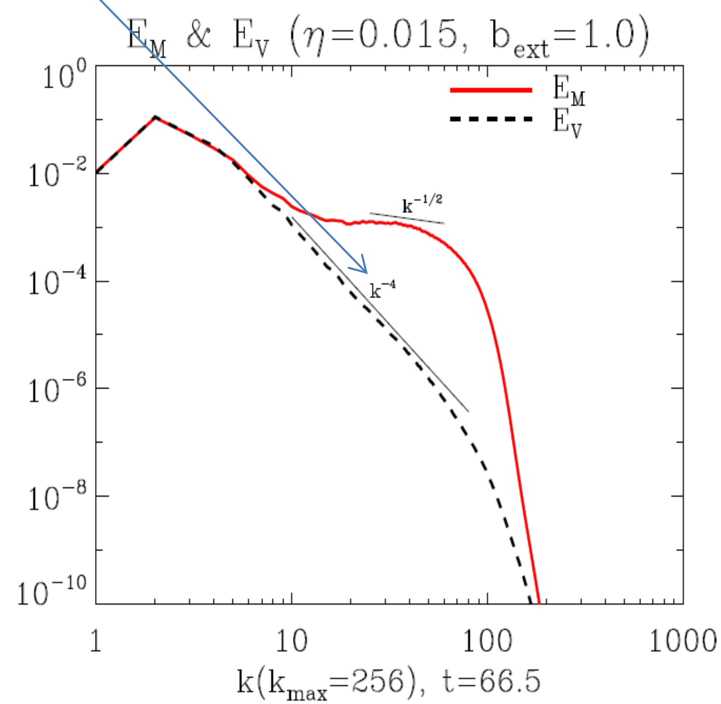
$$B\cdot\nabla B \sim \nu\nabla^2 V$$

2. **Magnetic energy transfer rate in stationary state**

3. **Results**

$$\begin{cases} Weak\ B_{ext}:\ E_V = k^{-3}, E_M = k^{-1} \\ Strong\ B_{ext}:\ E_V = k^{-4}, E_M = k^{-1/2} \end{cases}$$



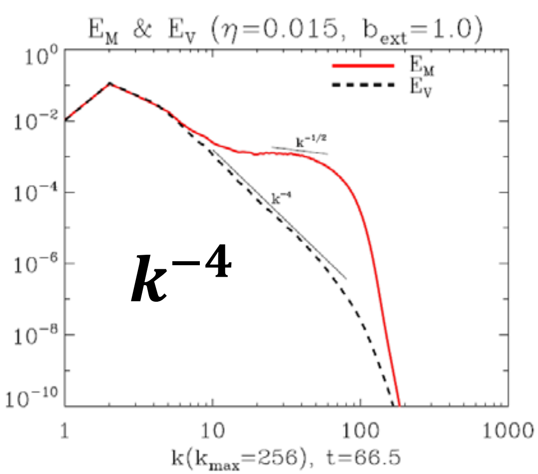



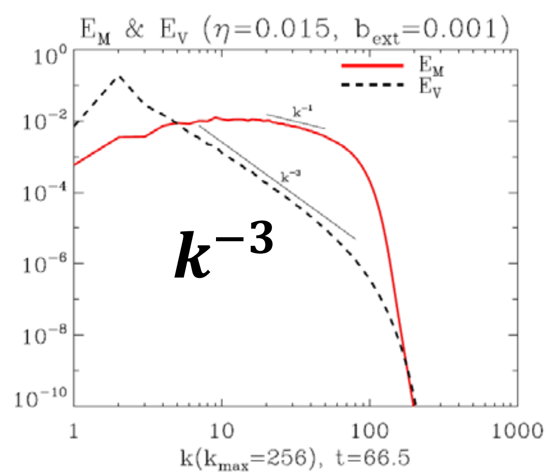
# Comparison

1. **Cho, Lazarian & Vishiniac 2003**
   – $E_V(\boldsymbol{k}^{-4})$ and $E_M(\boldsymbol{k}^{-1})$

2. **Schekochihin et al. 2004**
   – $E_V(\boldsymbol{k}^{-4})$ and $E_M(\boldsymbol{k}^{0})$

3. **Our results - scaling factor depends on $\mathbf{B}_{ext}$**
   – $E_V(\boldsymbol{k}^{-4})$ and $E_M(\boldsymbol{k}^{-1/2})$ with strong $\mathbf{B}_{ext}(1.0)$
   – $E_V(\boldsymbol{k}^{-3})$ and $E_M(\boldsymbol{k}^{-1})$ with weak $\mathbf{B}_{ext}(0.001)$



# Summary

1. MHD turbulence dynamo explains the evolution of magnetic fields
2. Magnetic energy cascades inversely (large scale dynamo) or migrates toward small scale (small scale dynamo)
   - mean field theory ($\alpha^2$model) & Kazantsev's model
3. Small scale dynamo in case of high $Pr_M$, which is suitable for warm and partially ionized galaxy, $E_M$ in subviscous scale is transferred to the kinetic eddies to extend viscous scale.
4. With balance relation and energy transfer rate,
   $E_V(\boldsymbol{k^{-4}} - \boldsymbol{k^{-3}})$ and $E_M(\boldsymbol{k^{-1}} - \boldsymbol{k^{-1/2}})$ depending on $\mathbf{B}_{\boldsymbol{ext}}$