MHD Turbulence in Expanding and Contracting Media

Junseong Park[1], Dongsu Ryu[1], and Jungyeon Cho[2]

[1]Dept. of Physics, Ulsan National Institute of Science and Technolog. Ulsan, Korea

[2]Dept. of Astronomy and Space Science, Chungnam National Univ. Daejeon, Korea

Purpose of Study

1. We investigated MHD turbulence by including the effects of expansion and contraction of background medium.

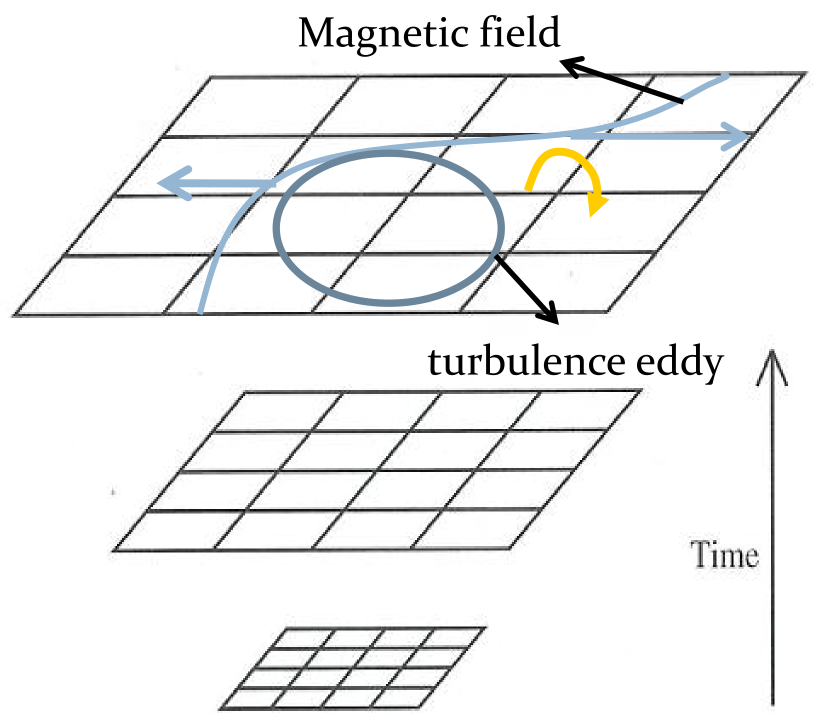
2. The main goal is to quantify the evolution and saturation of strength and characteristic lengths of magnetic fields in expanding and contracting media.

3. We examine the properties of turbulence in the regimes of $t_{eddy} < t_{exp\text{-}cntr}$ and $t_{eddy} > t_{exp\text{-}cntr}$. Based on it, we derive a scaling for the time evolution of rms peculiar velocity and magnetic field.

Contents

Fluid in expanding/contracting coordinate

Expansion in the comoving coordinate system



- comoving coordinate system

$$\mathbf{r} = a(t)\mathbf{x}$$

r is physical distance
x is comoving distance
a(t) is scale factor

$$\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a}$$

$\mathbf{u} = a\dot{\mathbf{x}} + \mathbf{x}\dot{a}$

Proper velocity

peculiar velocity ($\mathbf{v} = a\dot{\mathbf{x}}$)

- When the matter expand, the magnetic field in the matter is expand with comoving coordinate system.

The MHD equation in expanding/contracting media

$v' = \sqrt{\rho}v = v/(a\sqrt{a})$ is included density in peculiar velocity and **B** is magnetic field ,

p' is the pressure , $J = \nabla \times$ **B** is the current , ν is the viscosity, η is the magnetic diffusion. Where f is a random driving force.

$$\frac{\partial \mathbf{v}'}{\partial t} = \sqrt{a}\mathbf{v}' \times (\nabla \times \mathbf{v}') - \frac{5}{2}\frac{\dot{a}}{a}\mathbf{v}' + \sqrt{a}\mathbf{J} \times \mathbf{B} + \frac{1}{a^2}\nu\nabla^2\mathbf{v}' + \nabla p' + f$$

$$\frac{\partial \mathbf{B}}{\partial t} = \sqrt{a}\nabla \times (\mathbf{v}' \times \mathbf{B}) - 2\frac{\dot{a}}{a}\mathbf{B} + \frac{1}{a^2}\eta\nabla^2\mathbf{B}$$

Where $\rho \propto 1/a^3$

$$a(t) = \left(\frac{t}{t_{exp-cntr}} + 1\right)^{a_p}$$

Scale factor with $a_p = 1$ for expand, $a_p = -1$ for contract

$$\begin{cases} a_p = 1 & \Rightarrow \quad \dot{a} = 1/t_{exp} \\ a_p = -1 & \Rightarrow \quad \dot{a} = -t_{cntr}/(t + t_{cntr})^2 \end{cases}$$

$t_{exp-cntr}$ is smaller, the expanding and contracting rate increase.

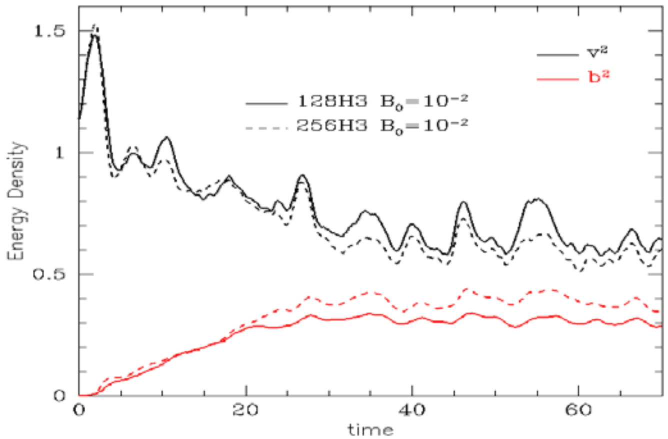
Simulation –initial condition

- Resolution : 256^3 grid (periodic box size = 2π)
- Incompressibility is assumed.
- Have considered only case of $\nu = \mu$
- Have considered hyperviscosity, hyperdiffusion
- At t=0, Mean magnetic field strength is $B_0 = 0.0001$
- Have simulated using t_{exp}=10, t_{cntr}=1

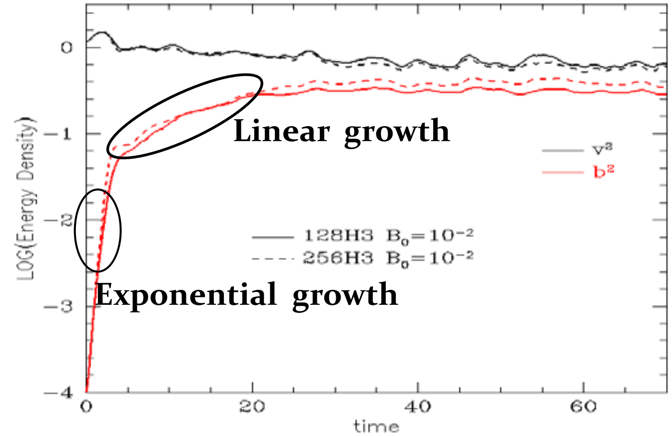
Table 1: Simulation conditions in the case of decaying turbulence

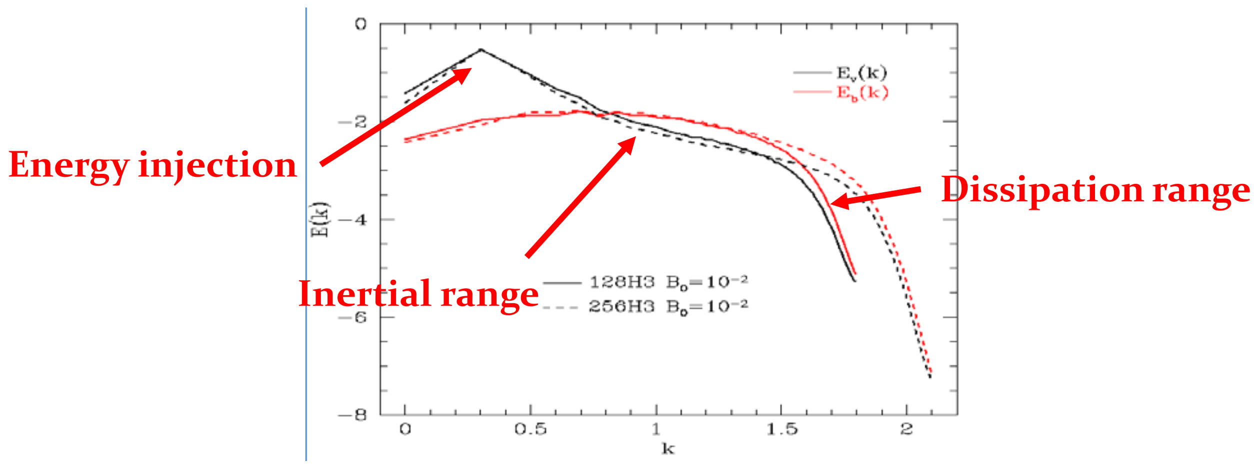
Resolution	Condition	$t_{exp-cntr}$	Strength of mean B_0 field
	contracting	1	0.0001
256^3	expanding	10	0.0001
	Not contracting/expanding	0	0.0001

The incompressible MHD turbulence without expanding/contracting effect



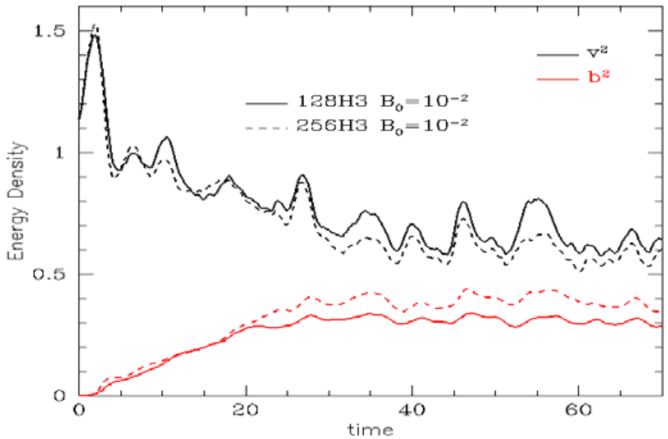
Logarithmic scale

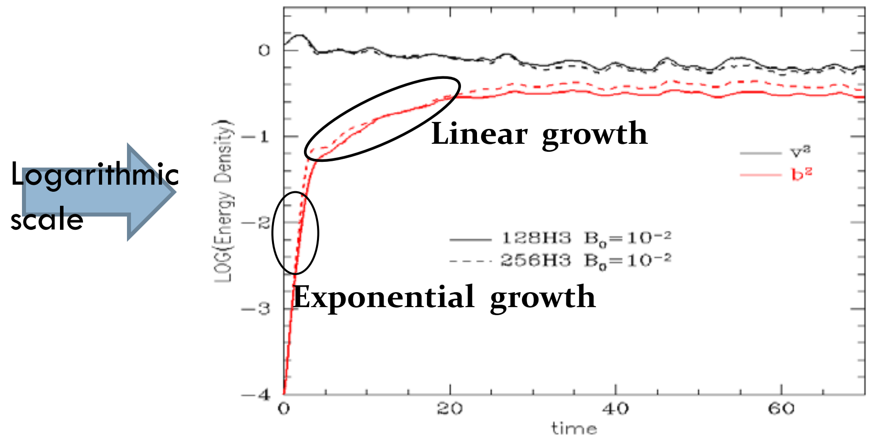




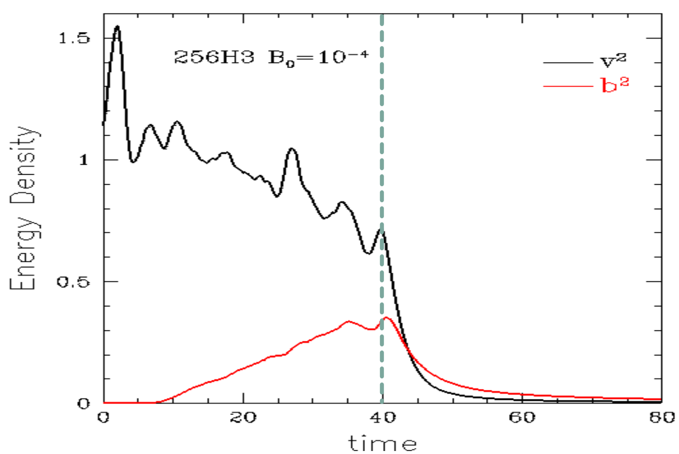
- The kinetic energy peak occurs at the energy injection scale. And magnetic energy spectrum peak occurs at larger wave number than the energy injection scale.
- Runs 128^3 and 256^3 resolution show similar growth rates and final saturation level.

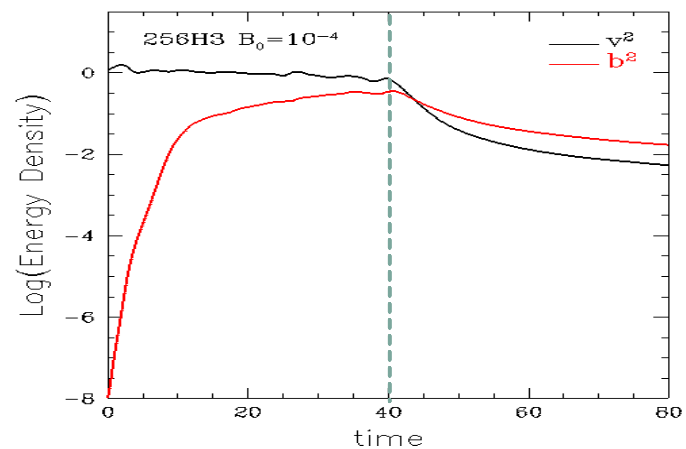
The incompressible MHD turbulence without expanding/contracting effect





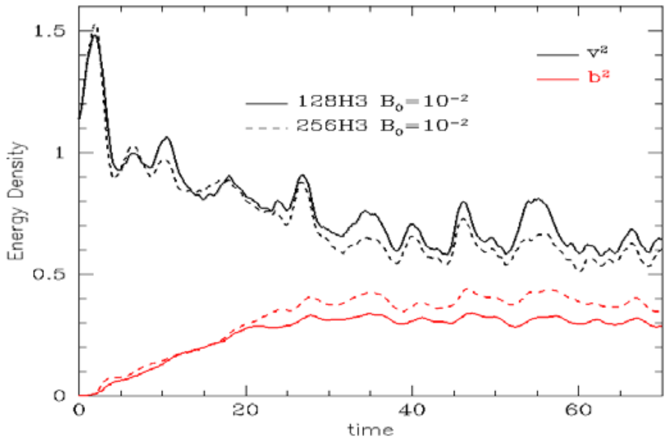
After that the turbulence has reached a stationary state, we turn off the random driving forces.



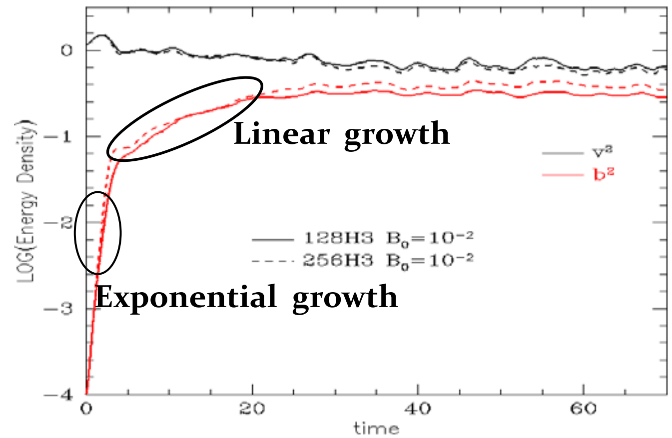


From this time (t=40) , we let the turbulence decay and inject the effect of the expansion/contraction.

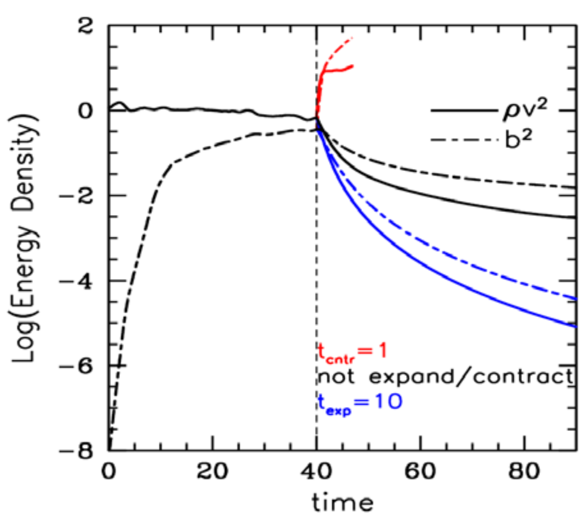
The incompressible MHD turbulence without expanding/contracting effect



Logarithmic scale



After that the turbulence has reached a stationary state, we turn off the random driving forces.

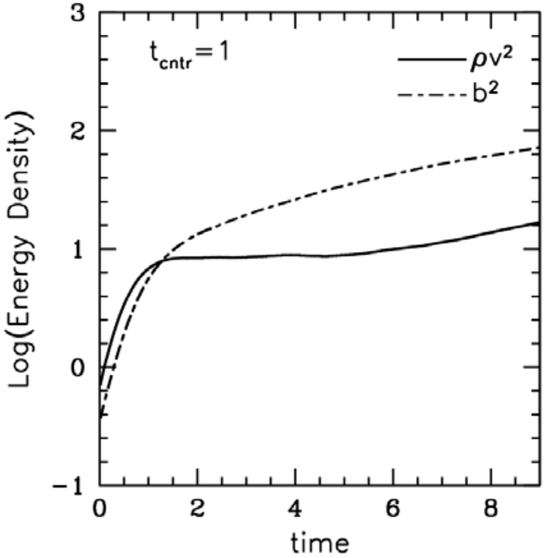


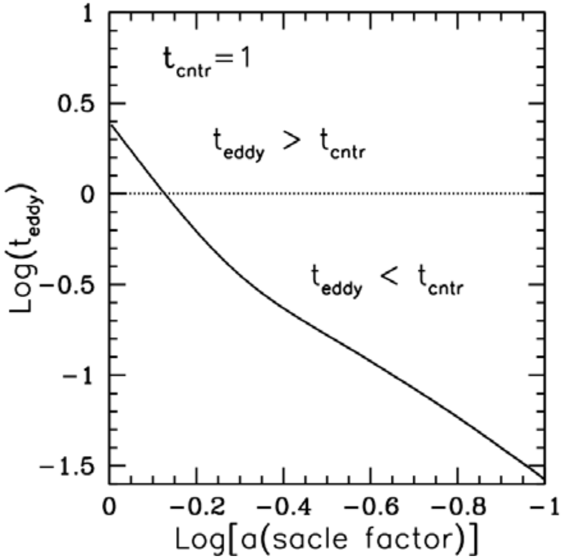
From this time (t=40) , we let the turbulence decay and inject the effect of the expansion/contraction.

Energy density and Eddy turn over time

Contracting media

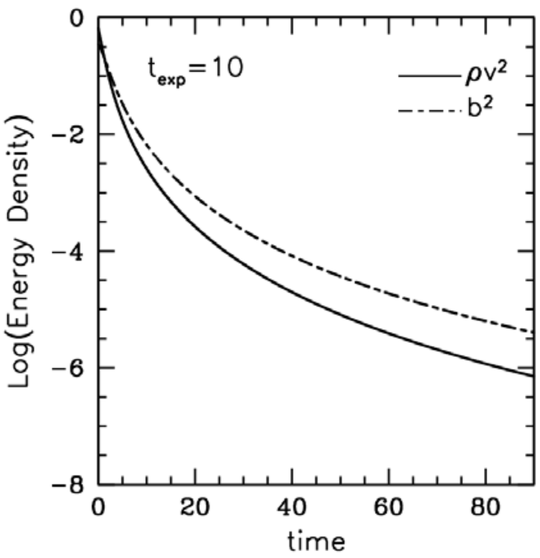
$$t_{eddy} = a L_0 / (\rho v^2 + b^2)^{1/2}$$

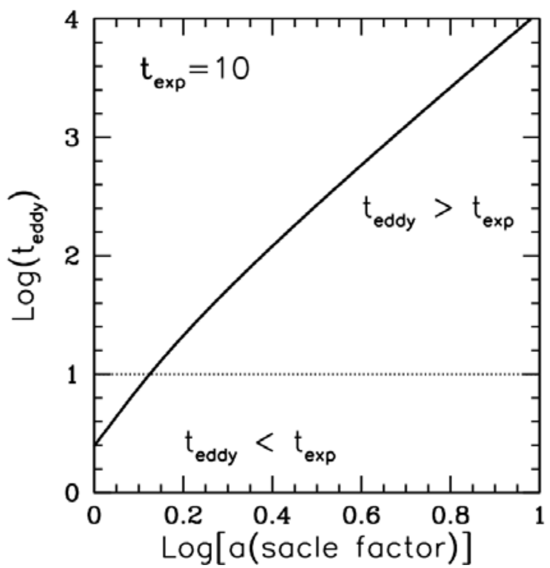




- $a = \left(\frac{t}{t_{cntr}} + 1\right)^{-1} \propto 1/t$
- t_{eddy} decrease with the time evolution.
- $t_{eddy} > t_{cntr}$ change to $t_{eddy} < t_{cntr}$ $a \approx -0.2$

Expanding media

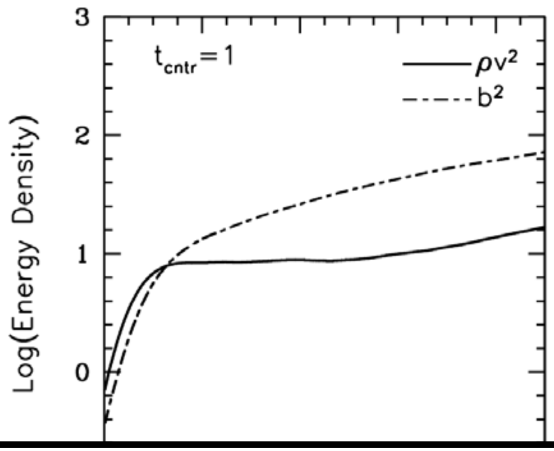


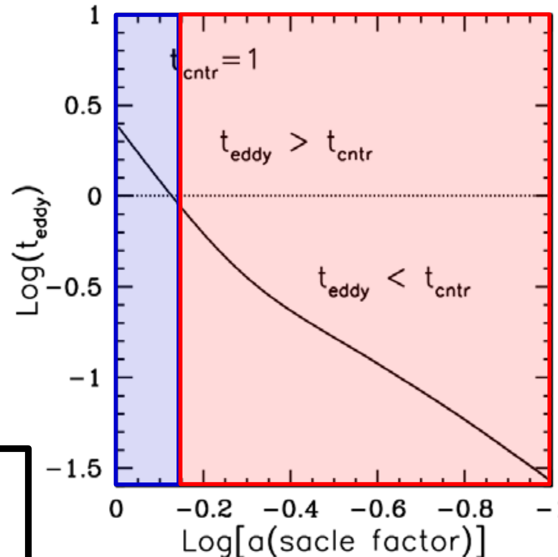


- $a = \left(\frac{t}{t_{exp}} + 1\right)^{1} \propto t$
- t_{eddy} increase with the time evolution.
- $t_{eddy} < t_{exp}$ change to $t_{eddy} > t_{exp}$ at $a \approx 0.2$

Energy density and Eddy turn over time

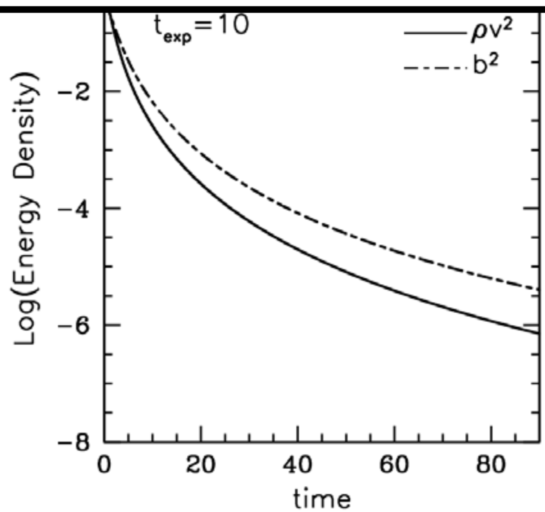
Contracting media

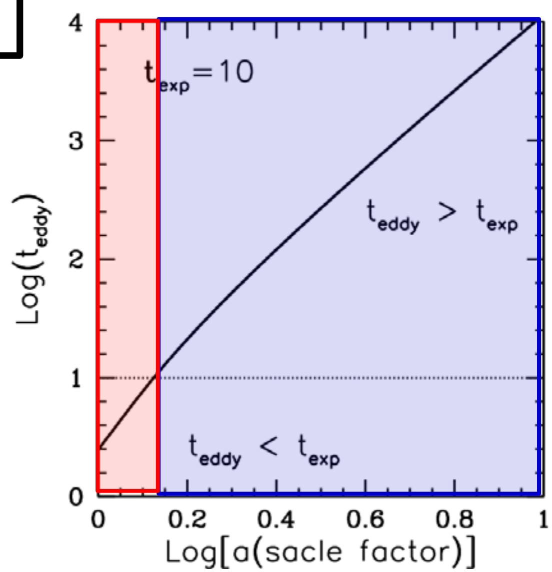




- t_{eddy} > t_{cntr} change to t_{eddy} < t_{cntr} $a \approx$ -0.2
- The contracting effect is initially dominant and then at this point it's changed to the evolution of turbulence is dominant.

Red zone
=> turbulence dominated
Blue zone
=> expanding
/contracting dominated



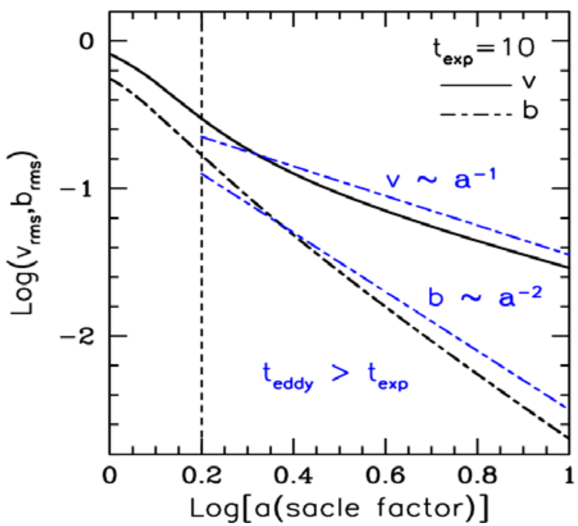


- t_{eddy} < t_{exp} change to t_{eddy} > t_{exp} at $a \approx$ 0.2
- The evolution of turbulence is initially dominant and then at this point it's changed to the expanding effect is dominant

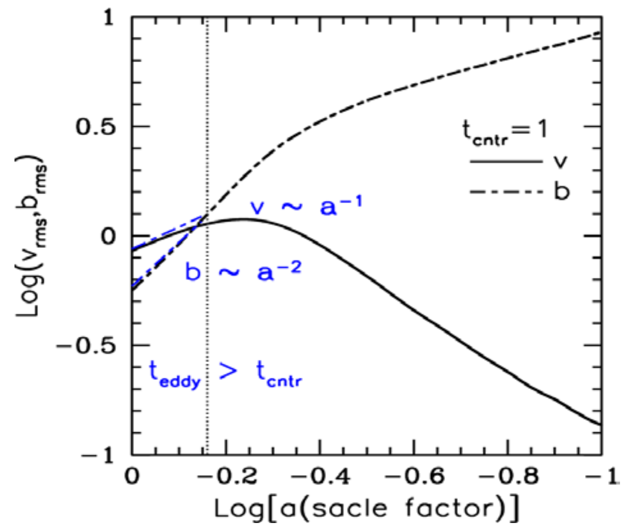
Scaling of velocity and magnetic field (t_{eddy} , t_{exp}, t_{cntr})

Effect of the expansion and contraction dominated

▣ Expanding media



▣ Contracting media



The MHD equation in expanding/contracting media

$$\frac{\partial \mathbf{v}'}{\partial t} = \sqrt{a}\,\mathbf{v}' \times (\nabla \times \mathbf{v}') - \frac{5}{2}\frac{\dot{a}}{a}\mathbf{v}' + \sqrt{a}\,\mathbf{J}\times\mathbf{B} + \frac{1}{a^2}\nu\nabla^2\mathbf{v}' + \nabla p' + f$$

$$\frac{\partial \mathbf{B}}{\partial t} = \sqrt{a}\,\nabla \times (\mathbf{v}' \times \mathbf{B}) - 2\frac{\dot{a}}{a}\mathbf{B} + \frac{1}{a^2}\eta\nabla^2\mathbf{B}$$

$$\frac{d\mathrm{v}}{dt} \sim -\frac{\dot{a}}{a}\mathrm{v} \;\Rightarrow\; \mathrm{v} \sim a^{-1}$$

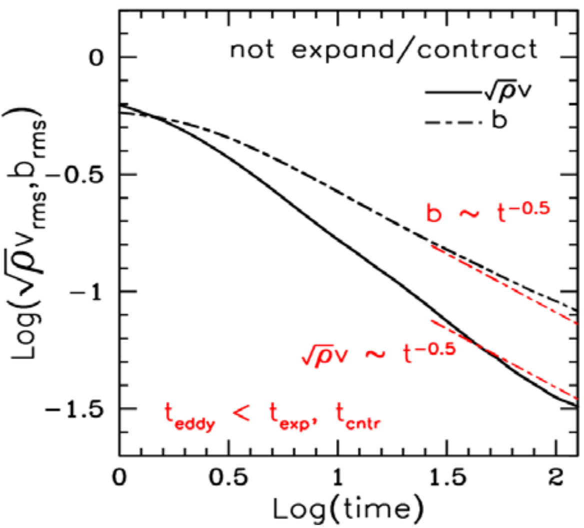
$$\frac{d\mathrm{B}}{dt} \sim -2\frac{\dot{a}}{a}\mathrm{B} \;\Rightarrow\; \mathrm{b} \sim a^{-2}$$

(Robertson & Goldriech 2012)

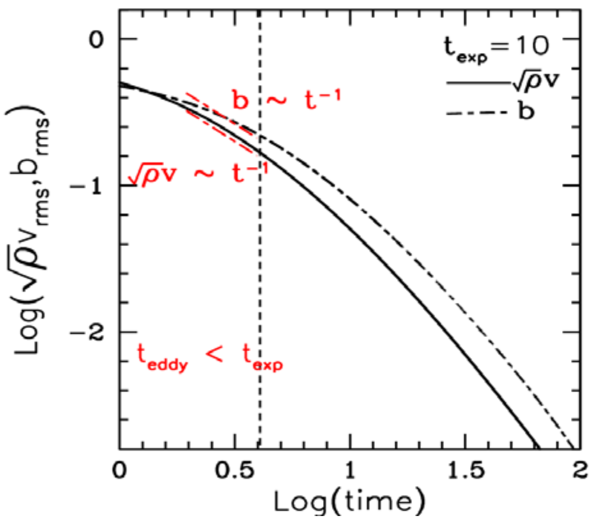
Scaling of velocity and magnetic field (t_{eddy} , t_{exp}, t_{cntr})

Turbulence dominated

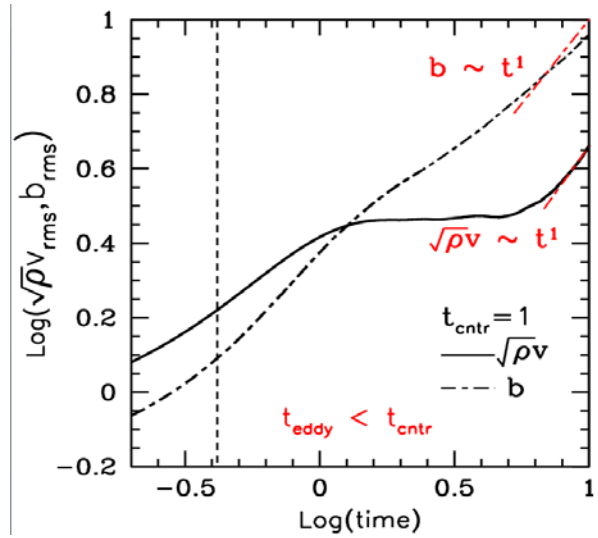
▣ **Not expand/contract**



▣ **Expanding media**



▣ **Contracting media**



(Left) v, b~$t^{-0.5}$ in decaying MHD turbulence regime
(Biskamp & Müller 1999)

(Middle, Right) $\sqrt{\rho}$v, b~t^{-1} for expanding media, $\sqrt{\rho}$v, b~t^{1} for contracting media

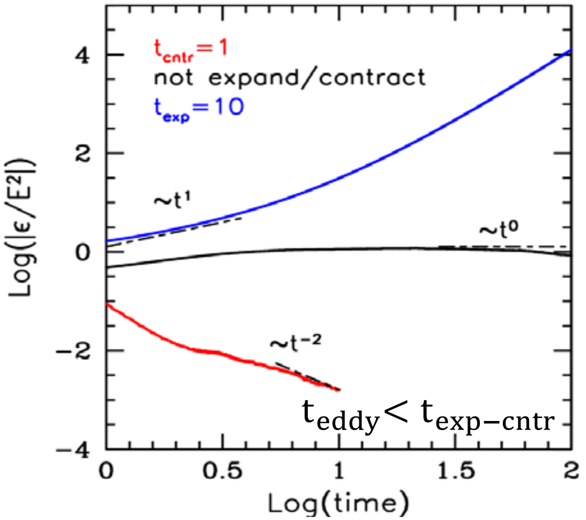
$$\frac{d\log(\omega/H)}{d\log(1/a)} = \left(2 + \eta\frac{\omega}{H}\right) - \frac{d\log H}{d\log(1/a)}$$

(Robertson & Goldriech 2012)

$\omega \sim 1/t_{eddy}$, $\eta = 1.2$

$H = \dot{a}/a$

Energy decay rate for Turbulence dominant case

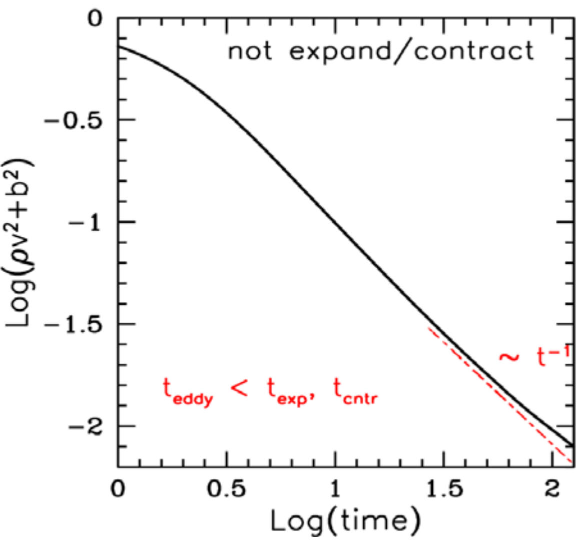


Energy decay rate: $\epsilon = -\frac{dE}{dt}$, E= $E^K + E^M$

$(dE/dt)E^{-2}$ = CONST ⇒ $E \sim t^{-1}$

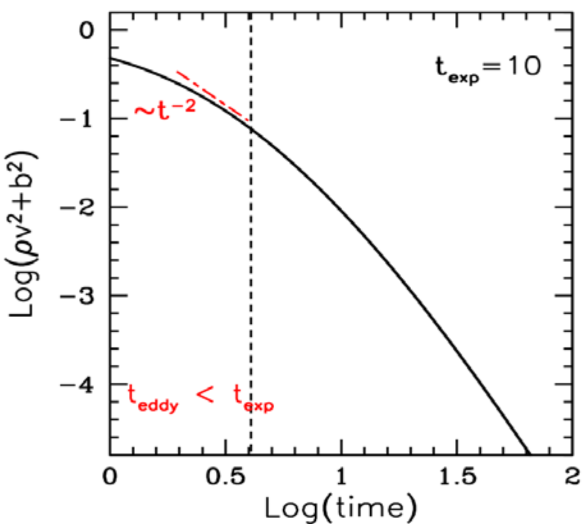
in decaying MHD turbulence regime
(Biskamp & Müller 2000)

▣ Not expand/contract



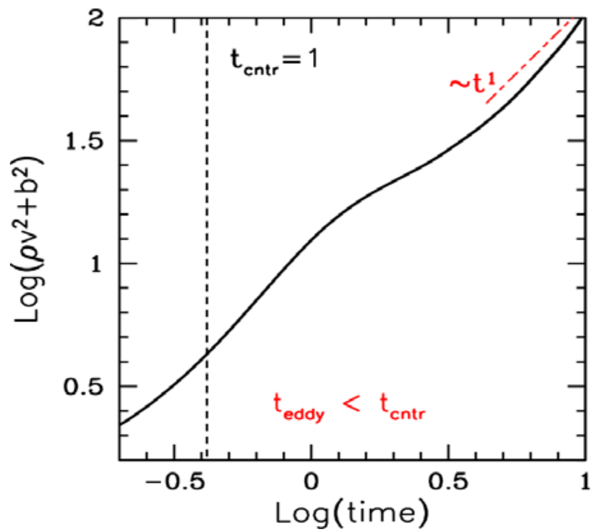
$E \sim t^{-1}$

▣ Expanding media



$E \sim t^{-2}$

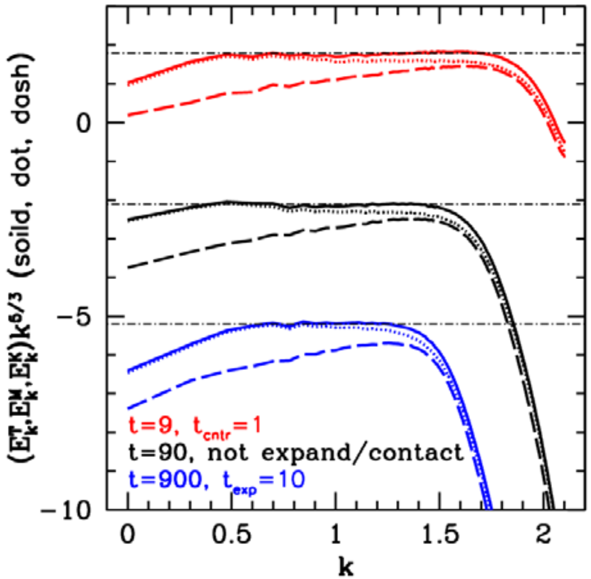
▣ Contracting media



$E \sim t^1$

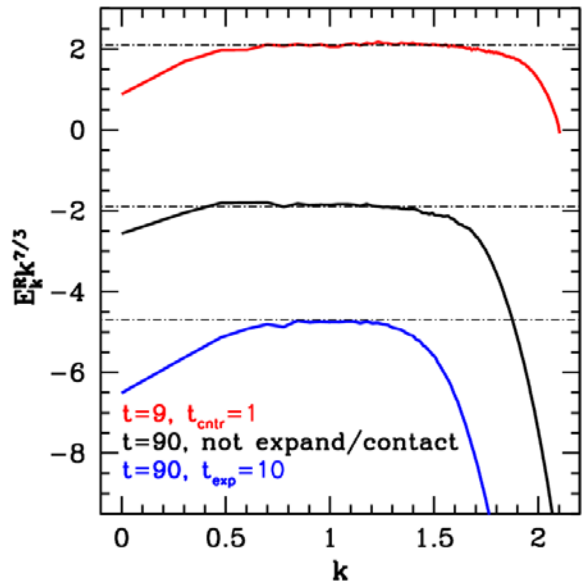
Energy spectrum

▣ Total energy spectrum



$$E_k^T = E_k^{\mathrm{K}} + E_k^{\mathrm{M}}$$

▣ Residual spectrum



$$E_k^R = E_k^{\mathrm{M}} - E_k^{\mathrm{K}}$$

- The dissipation range moves toward (larger wave number in contracting media
  smaller wave number in expanding media

- Regardless of the expansion and contraction effects, the total energy and residual spectrum follows the $E_k^T \sim k^{-5/3}$ and $E_k^R \sim k^{-7/3}$ (Müller 2005) in the inertial range.

Conclusion

- We performed a preliminary study of incompressible MHD decaying turbulence by including the effect of expansion and contraction.

- Scaling for velocity and magnetic field in expanding/contracting media

  $t_{eddy} > t_{exp-cntr}$ ⇒ $v \sim a^{-1}, b \sim a^{-2}$

  $t_{eddy} < t_{exp-cntr}$ ⇒ $\sqrt{\rho}v, b \sim t^{-1}$ in expanding media; $\sqrt{\rho}v, b \sim t^{1}$ in contracting media

- The total energy and residual spectrum follows the $E_k^T \sim k^{-5/3}$ and $E_k^R \sim k^{-7/3}$ (Müller 2005) in the inertial range.

- The specific results would depend on t_{eddy} and $t_{exp\text{-}cntr}$. We will explore those in future.