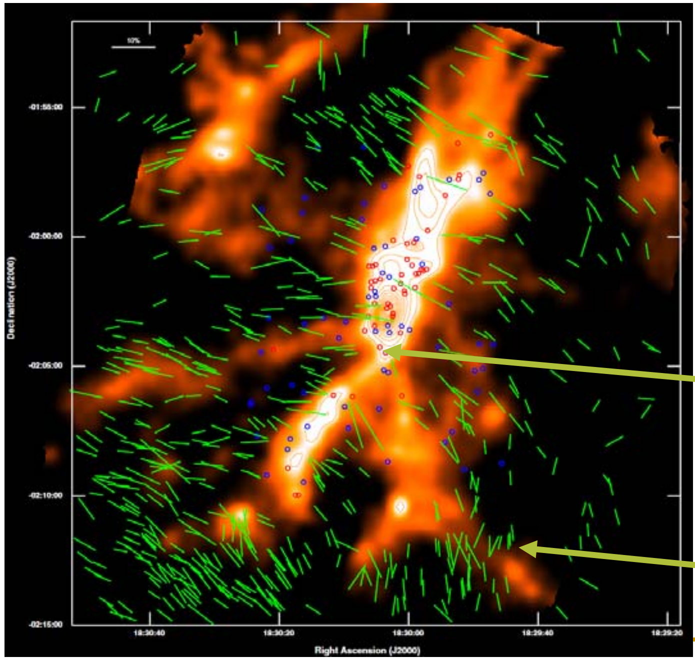
Magnetohydrostatic Equilibria of Isothermal Filamentary Clouds

Kohji Tomisaka（National Astronomical Observatory of Japan）

Ref. Structure and Mass of Filamentary Isothermal Cloud Threaded by Lateral Magnetic Field, 2014, ApJ, **785**, 24(12pp)

Filamentary Cloud

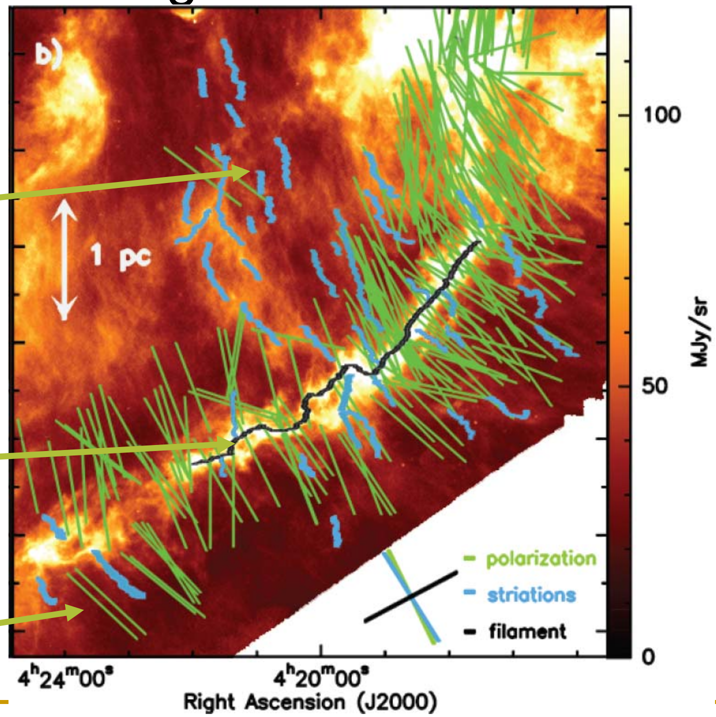
- *Herschel* (mid- far-IR obs.) has revealed many filaments in thermal dust emissions. Filaments are regarded as basic building blocks of clouds.
- Near IR polarization observations indicate
  - Interstellar magnetic field is $\perp$ to the filaments with large column-density.
  - low column-density filament is extending $||$ to B.



Less-dense filaments with small σ

Dense filaments with large σ

IS B-field



Serpens South Cloud by Sugitani et al (2011).

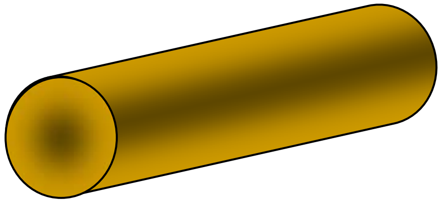
Taurus Cloud (B211/213) by Palmeirim et al. (2013).

Equilibria of Isothermal Filamentary Clouds

- No Magnetic Field (Stodolkiewicz 1963; Ostriker 1964)

$$\rho(r) = \rho_c \left(1 + \frac{r^2}{8H^2}\right)^{-2}$$

Scale-height $H = c_s / (4\pi G \rho_c)^{1/2}$



- Line-mass

$$\lambda(R) \equiv \int_0^R 2\pi r \rho(r) dr = \frac{2c_s^2}{G} \frac{R^2/8H^2}{1+R^2/8H^2} \leq \frac{2c_s^2}{G}$$

- Max. line-mass

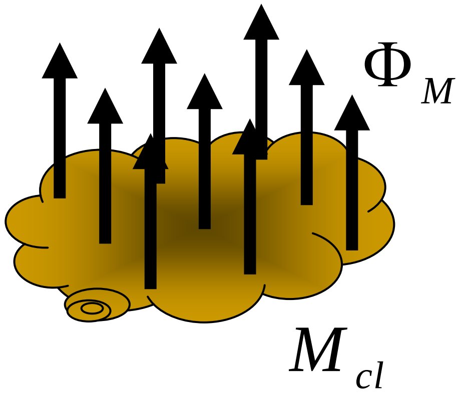
$$\lambda_{\max} = \frac{2c_s^2}{G}$$

$\lambda > \lambda_{\max}$ ➔ No equilibria ➔ dyn. contraction

$\lambda < \lambda_{\max}$ ➔ equilibrium solution with a finite density-contrast

Magnetic Field

- Magnetic field controls the stability of clouds



$M_{cl} > M_{crit}$ Magnetically Supercritical Clouds
➔ dynamical contraction

$M_{cl} < M_{crit}$ Magnetically Subcritical Clouds
➔ Magnetohydrostatic state evolves quasistatically by magnetic (ambipolar) diffusion

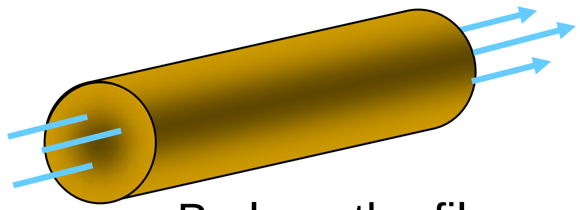
Magnetically Critical Mass

$$M_{crit} \simeq \Phi_{Mag} / 2\pi G^{1/2}$$

when $M_{crit} >> M_J$

This is for a 3D cloud.
How about a filamentary cloud?

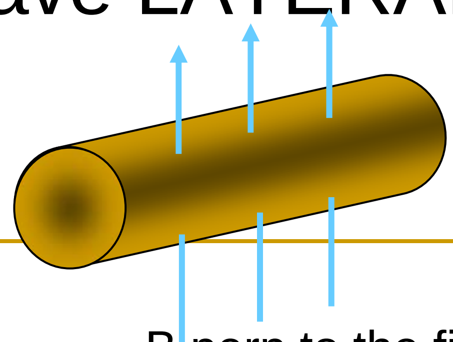
Magnetized Filaments



- Model with constant plasma β
  $(\beta \equiv p/(B_z^2/8\pi))$ (Stodolkiewicz 1963)

$$\lambda = \frac{2c_s^2}{G}(1+\beta^{-1})\frac{R^2/8H^2}{1+R^2/8H^2} \qquad H = \frac{c_s(1+\beta^{-1})}{(4\pi G\rho_c)^{1/2}}$$

- Model with a constant mass/flux ratio
  ($\phi \equiv \rho / B_z$ is conserved in the radial contraction)
  (Fiege & Pudritz 2000a,b)
  - Line-mass increases with B-field strength.
- However, observed filaments have LATERAL B-field.



Method to Obtain Magnetohydrostatic Equilibria of Isothermal Filament

- Basic equations → Force-balance, Ampere's law, Poisson eq.

Grad-Shafranov Eq. of flux function Φ(x,y)

$$\nabla^2 \Phi = -\frac{1}{2}\frac{dq(\Phi)}{d\Phi}\exp(-\psi), \qquad \mathbf{B} = \nabla \times (\Phi \mathbf{e}_z)$$

Poisson Eq. of grav. pot.

$$\nabla^2 \psi = q(\Phi)\exp(-\psi), \qquad \mathbf{g} = -\nabla \psi$$

$$q(\Phi) = \frac{d\lambda / d\Phi}{2\int_0^{y_s(\Phi)} \exp(-\psi)/(\partial\Phi/\partial x)_y \, dy},$$

Mass-Loading: Mass distributior against magnetic flux.

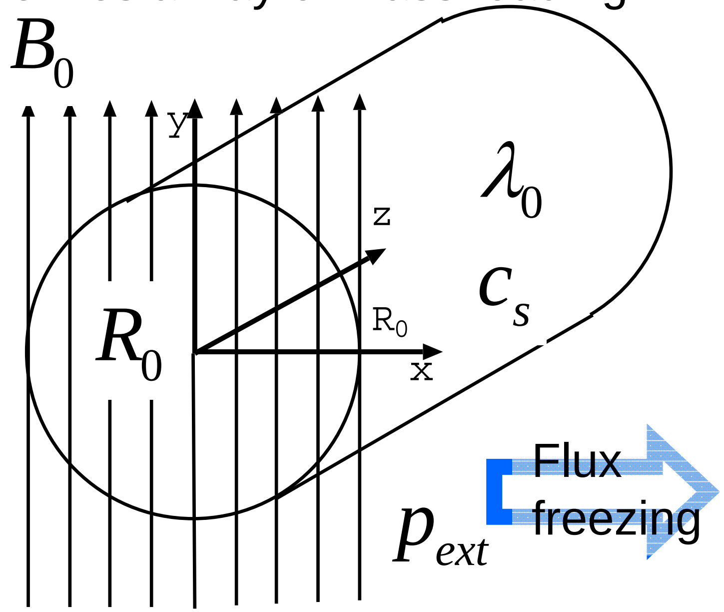
- Solve this simultaneous differential eq. by self-consistent-field method.

(Mouschovias 1976; Tomisaka+ 1988)

Parameters to Specify a Magnetohydrostatic Equilibrium

"Parent" filament

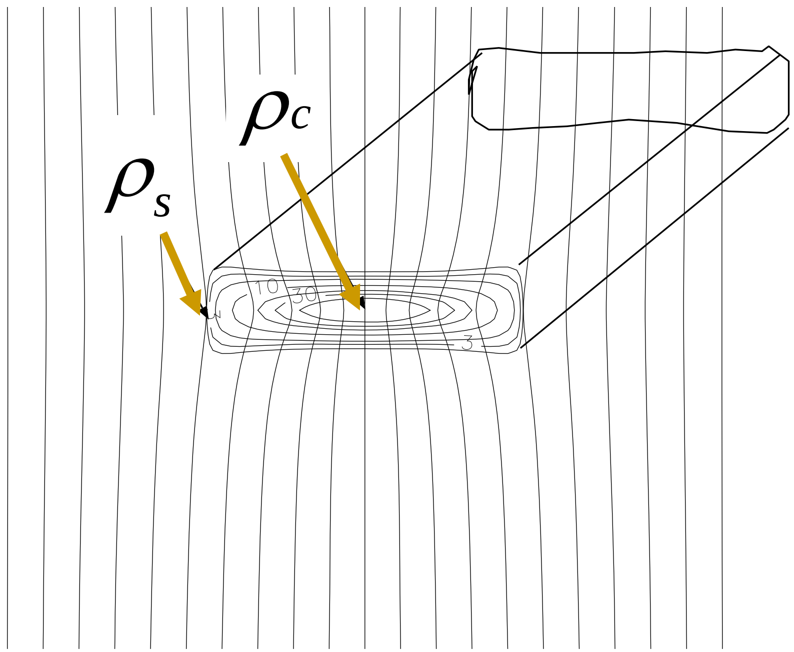
defines a way of mass-loading



We consider a cylinder
with a uniform density,
a radius R_0, a uniform B-field B_0
and sound speed c_s is immersed in
external pressure p_{ext}.

Equilibrium in balance b/w

gravity,Lorentz force, and thermal pressure



Thin and wide noodle

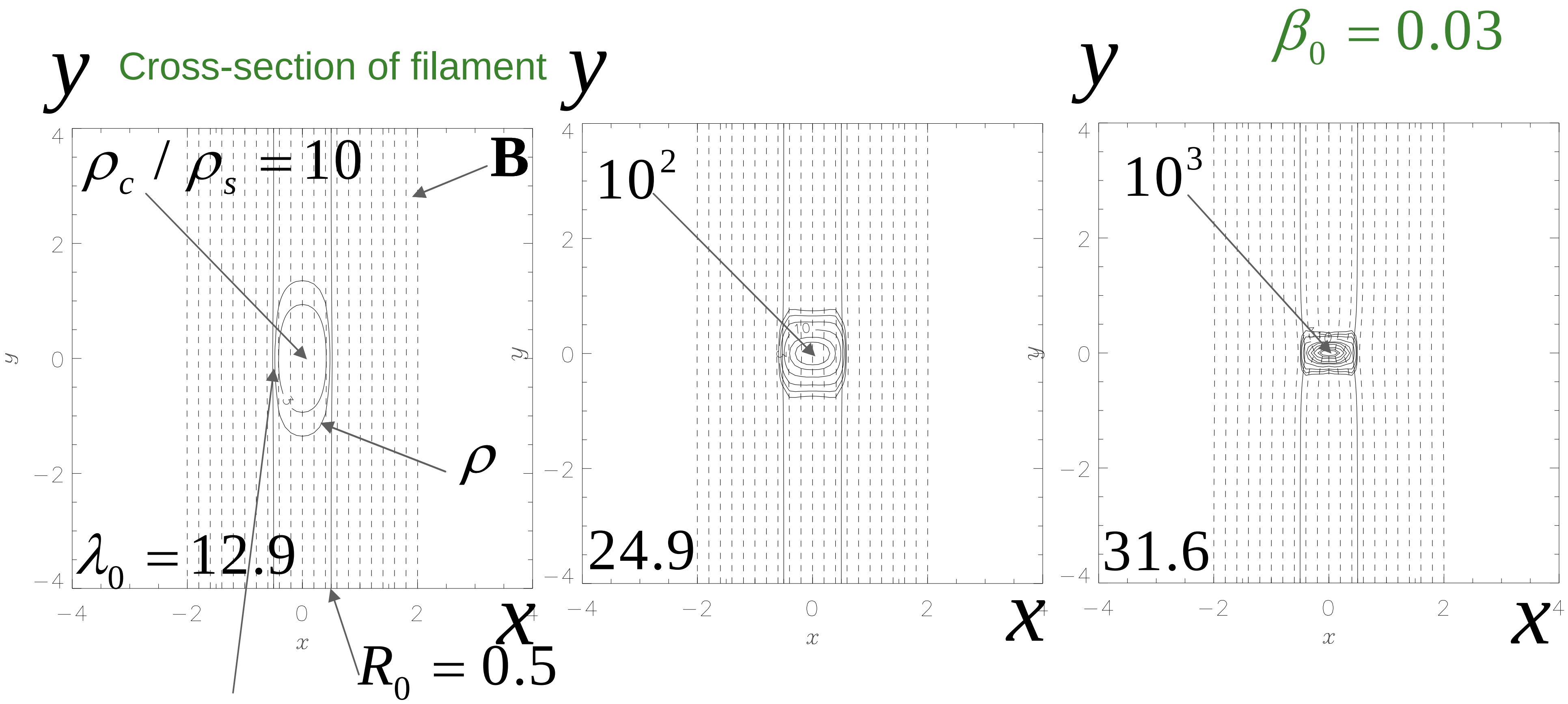
density at the surface $\rho_s = p_{ext} / c_s^2$

central density ρ_c

After normalization, the problem contains 3 parameters:

Density contrast	Ambient plasma β	Radius of "Parent" filament
ρ_c / ρ_s	$\beta_0 \equiv p_{ext} / (B_0^2 / 8\pi)$	$R_0 / [c_s / (4\pi G \rho_s)^{1/2}]$

Result 1　Small R_0=0.5 of Parent Cloud

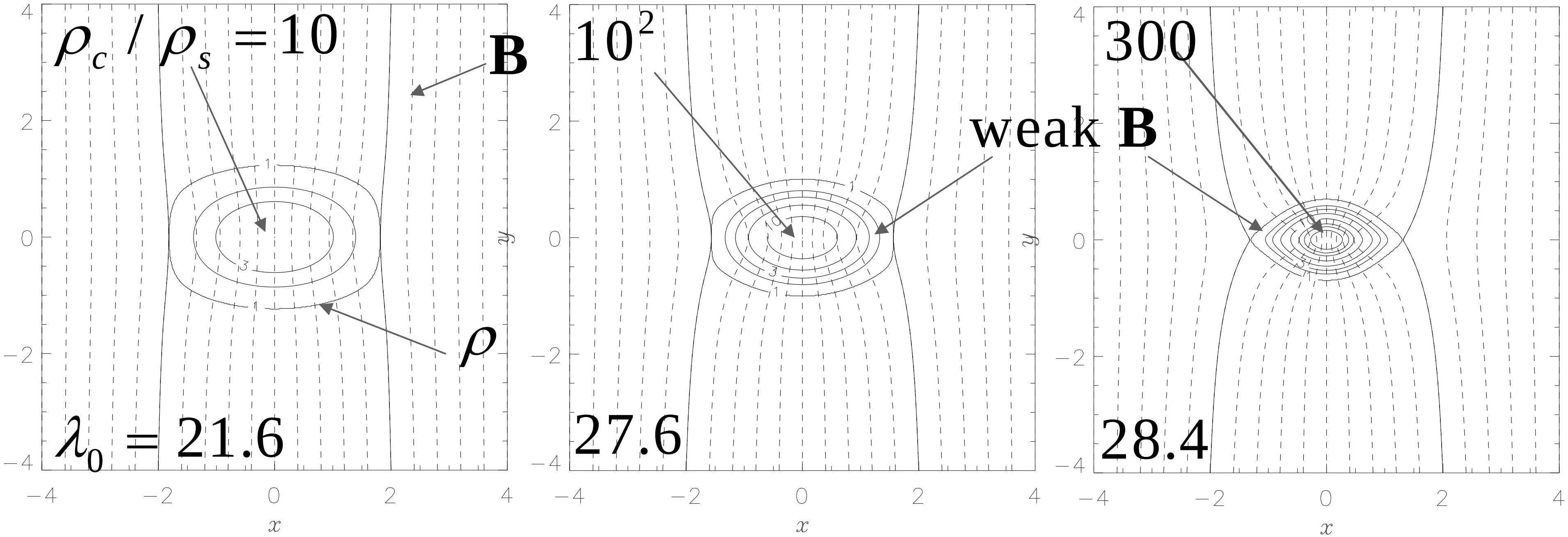


B-field bows outwardly. Magnetic confinement.

(1) Line-mass λ_0 increases with central density ρ_c.
(2) The filament with low ρ_c extends along B-field.
(3) That with high ρ_c has a major axis perp to B-field.

Result(2) Standard Model $(R_0 = 2, \beta_0 = 1)$

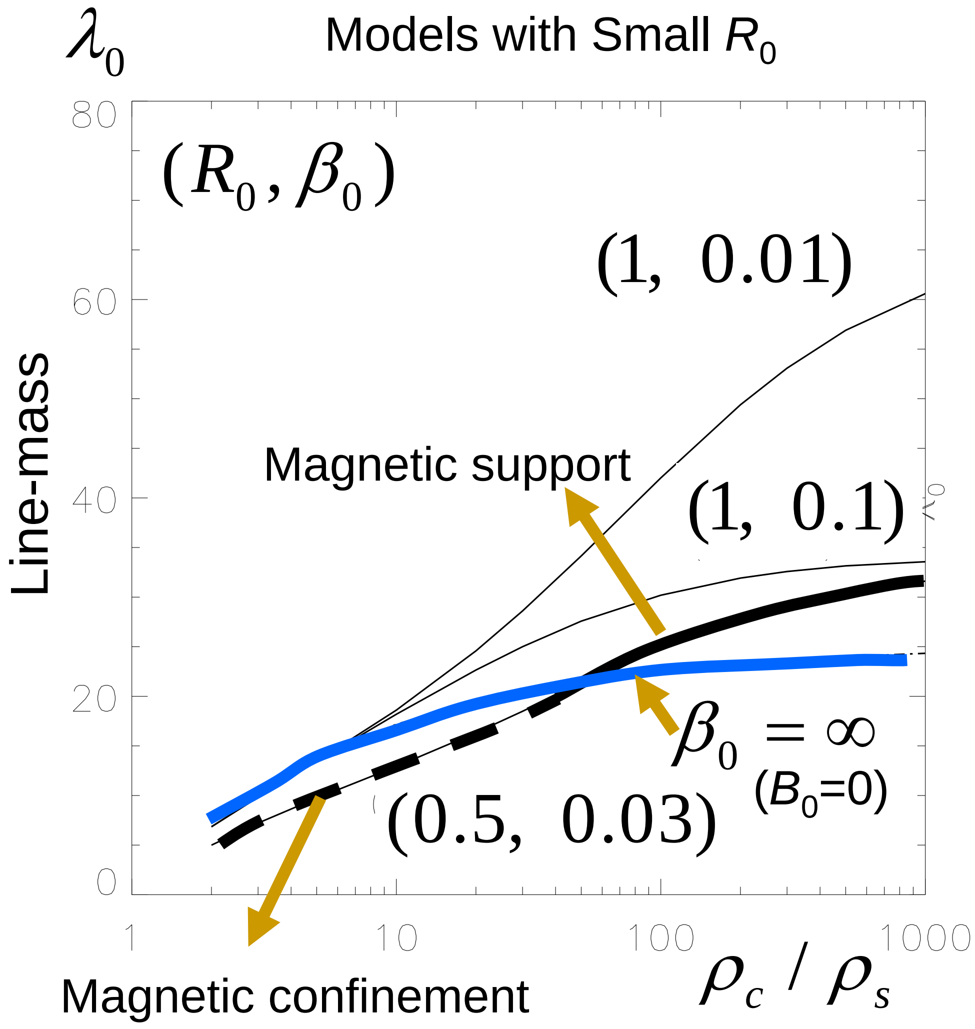
Cross-section of filament

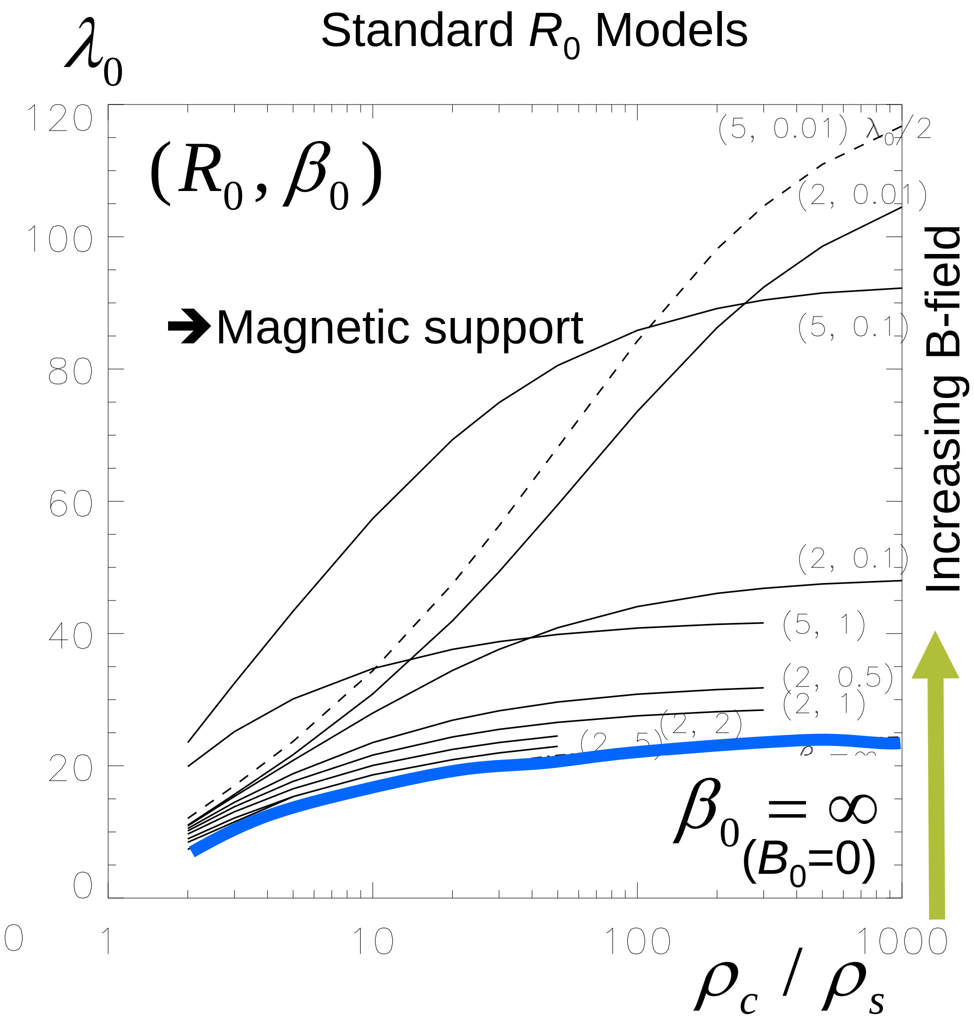


Hour-glass type B-field.

(1) Line-mass λ_0 increases with central density ρ_c.
(2) The major axis is elongated perp to B-field.
(3) Regions of weak B-field are found near the equator.

Central Density ρ_c Line-Mass λ_0 Relation





In special cases, B-field reduces λ_0.

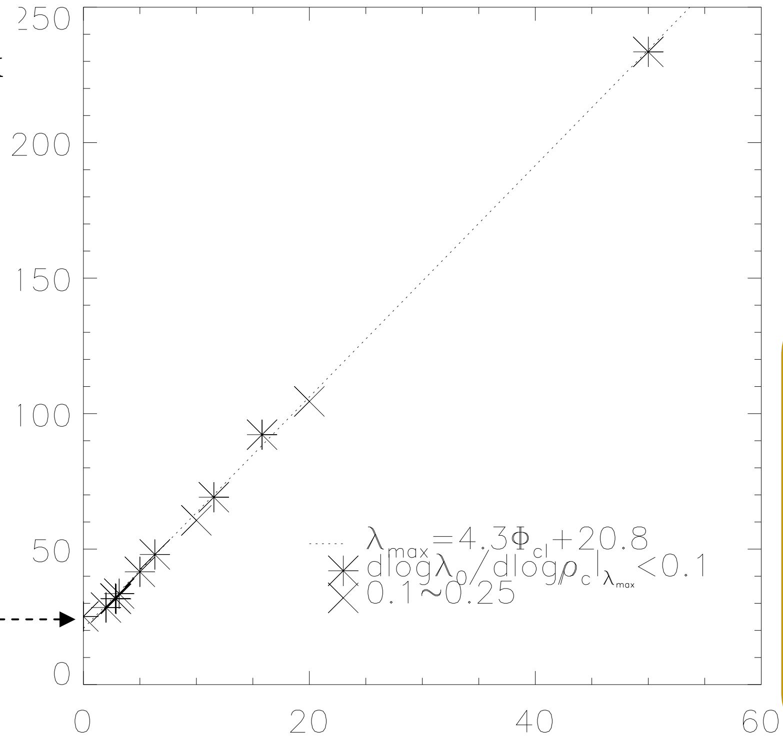
B-Field supports the filament

Critical Line-Mass of the Filament

nomarized $\lambda_{\max}$

Max. line mass

8π



normalized $\Phi_{cl} = R_0 B_0$

（Mag. flux per unit length）

Least Squares Method

$$\lambda_{\max} \approx 0.24\Phi_{cl} / G^{1/2}$$

dimensional

$$+1.66 c_s^2 / G$$

When the magnetic flux exceeds

$$\Phi_{cl} = R_0 B_0 > 3\mu\text{G pc}$$

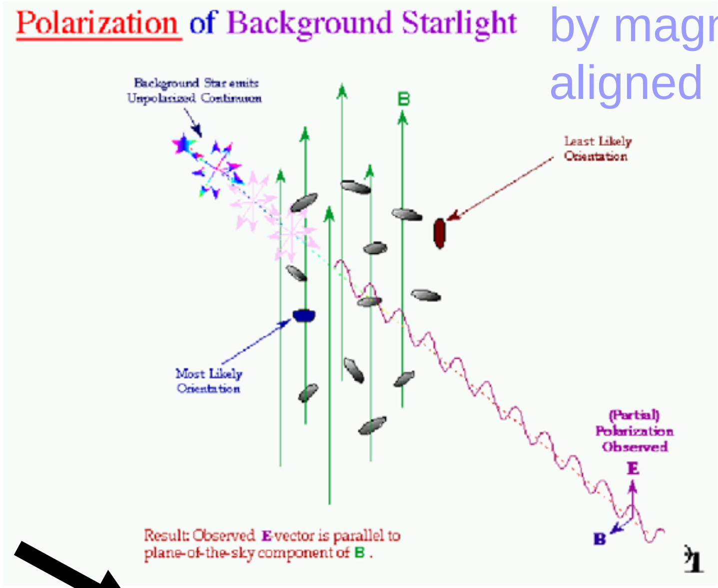
maximum line-mass is determine
by the magnetic flux per length.

Take notice of the similarity
to the mass formula for a thin
disk $M_{\max} \approx \Phi_{cl} / 2\pi G^{1/2}$

(2) Observational Expectation of Polarization

(1) extinction

$$\mathbf{B} \,//\, \mathbf{E}$$

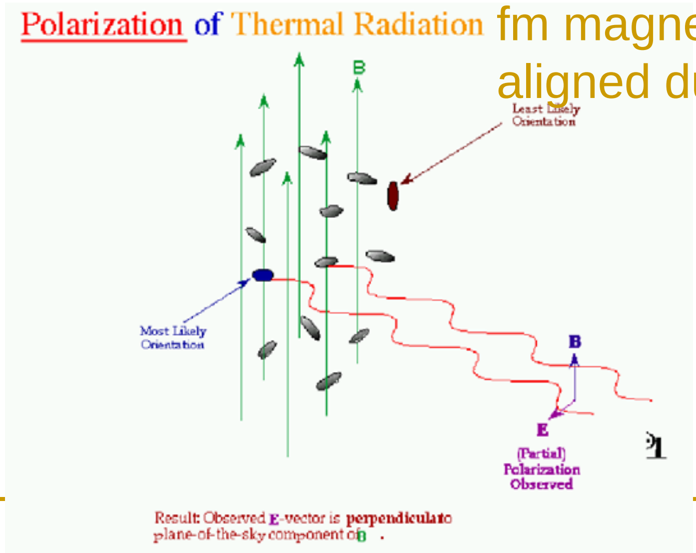


by magnetically aligned dusts

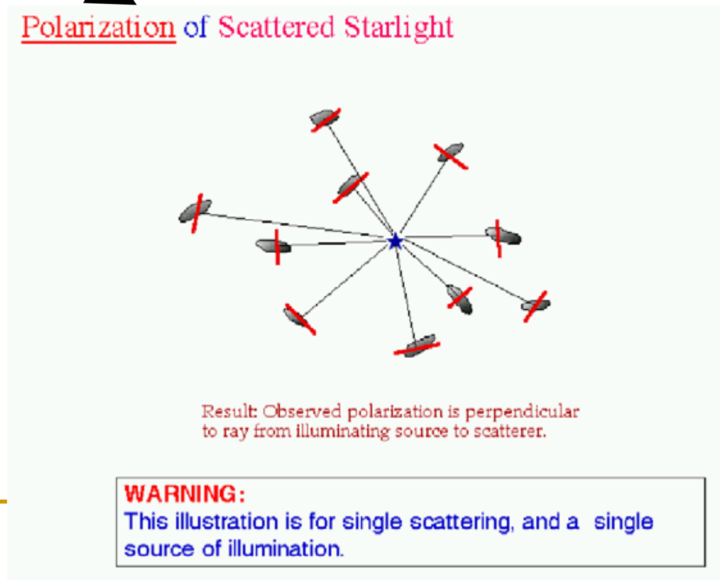
(2) thermal dust emission

$$\mathbf{B} \perp \mathbf{E}$$

(3) scattering $\mathbf{E} \perp \mathrm{ray}$



fm magnetically aligned dusts



Polarization of Thermal Dust Emissions from oblate/prolate dusts aligned in the B-field direction.

$$Q = \int C \cdot R \cdot F \cdot c \cdot B_\nu(T) \rho \cos 2\psi \cos^2 \gamma ds$$

$$U = \int C \cdot R \cdot F \cdot c \cdot B_\nu(T) \rho \sin 2\psi \cos^2 \gamma ds$$

(Draine & Lee 85, Fiege & Pudritz 2000)

C: difference of cross sections perp and parallel to B

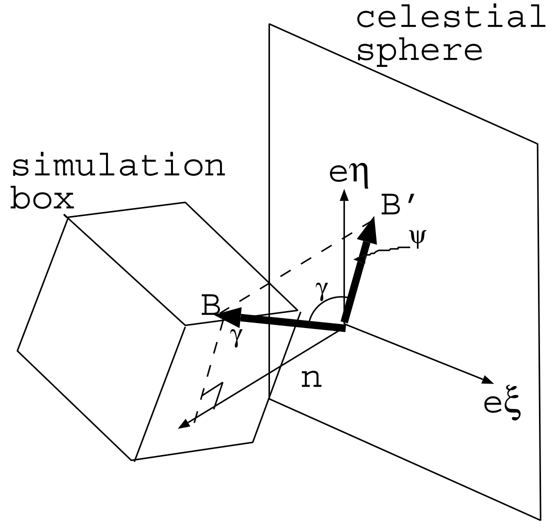
R: reduction factor due to imperfect grain alignment

F: reduction factor due to turbulent B-field

$c = \rho / n_d$

γ: angle b/w B and plane of the sky.

ψ: angle b/w projection of B and η-axis



Relative Stokes parameter (Wardle & Konigl 90)

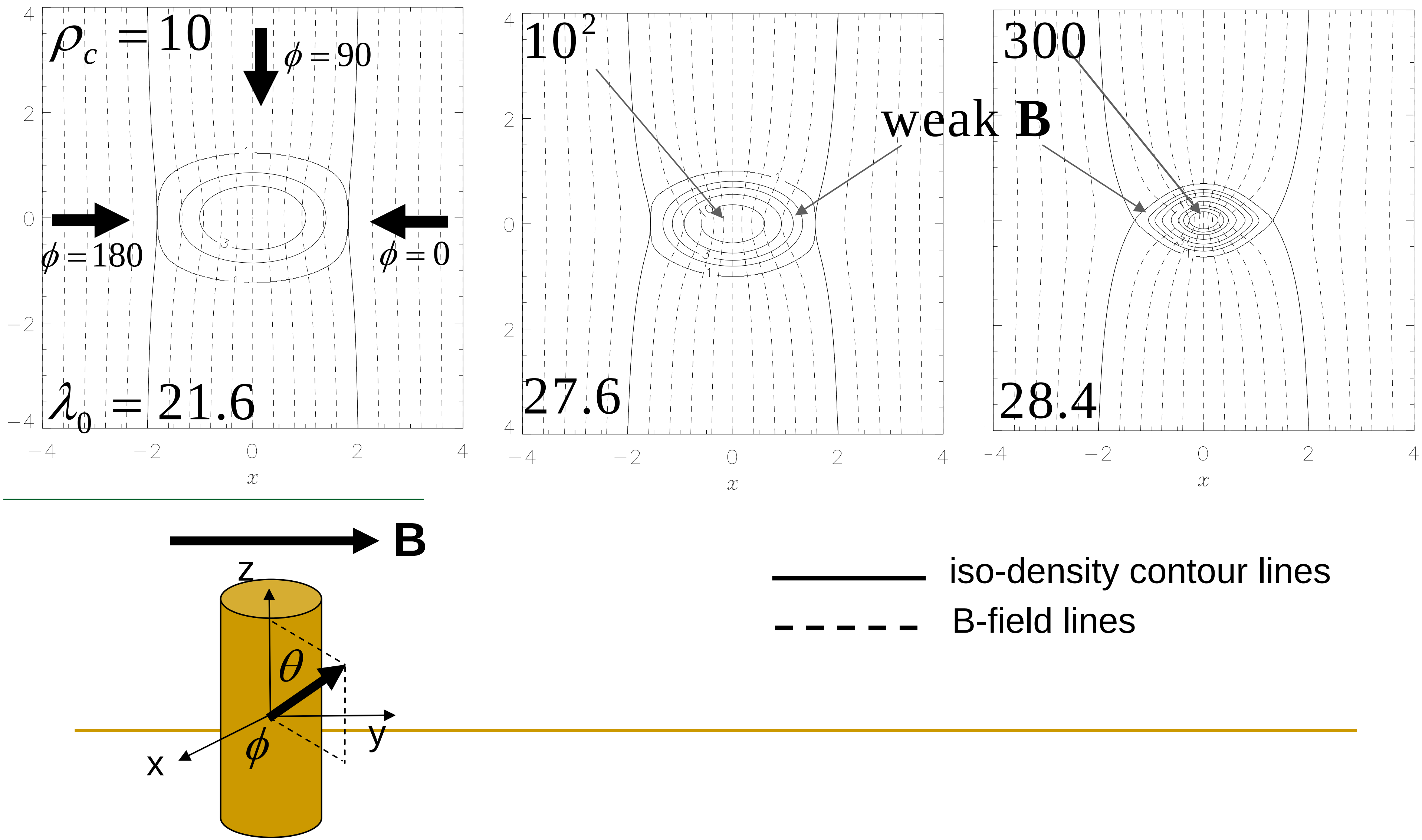
$$q = \int \rho \cos 2\psi \cos^2 \gamma ds$$

$$u = \int \rho \sin 2\psi \cos^2 \gamma ds$$

$$i = \int \rho ds$$

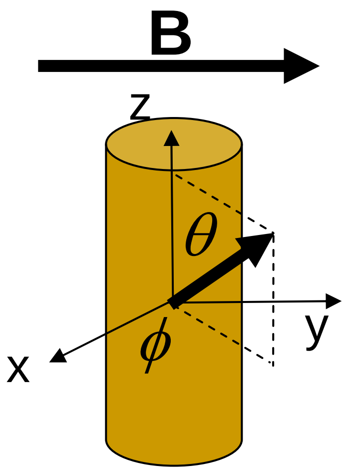
→ Polarization angle and polarization degree

Structure of Fiducial Model $(R_0 = 2, \beta_0 = 1)$

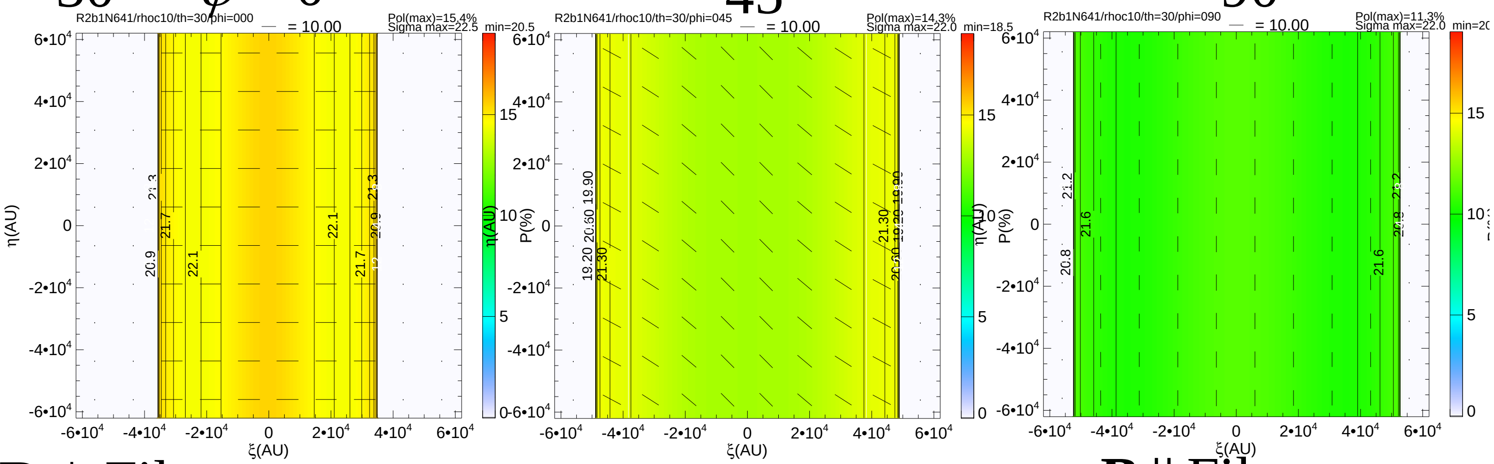


Expected Polarization (Thermal Dust Emissions)

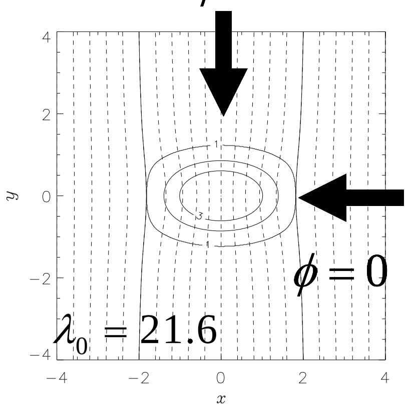
Models with Low Central Density $\rho_c = 10\rho_s$



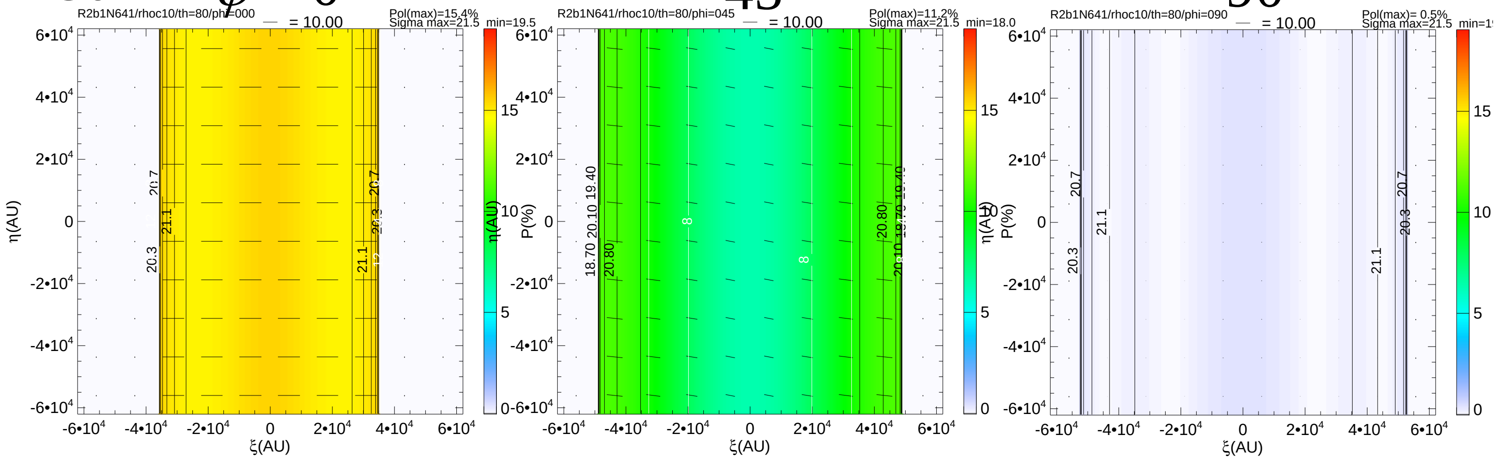
$\theta = 30^\circ$ $\varphi = 0^\circ$ 45° 90°



B ⊥ Filament → **B** || Filament



$\theta = 80^\circ$ $\varphi = 0^\circ$ 45° 90°

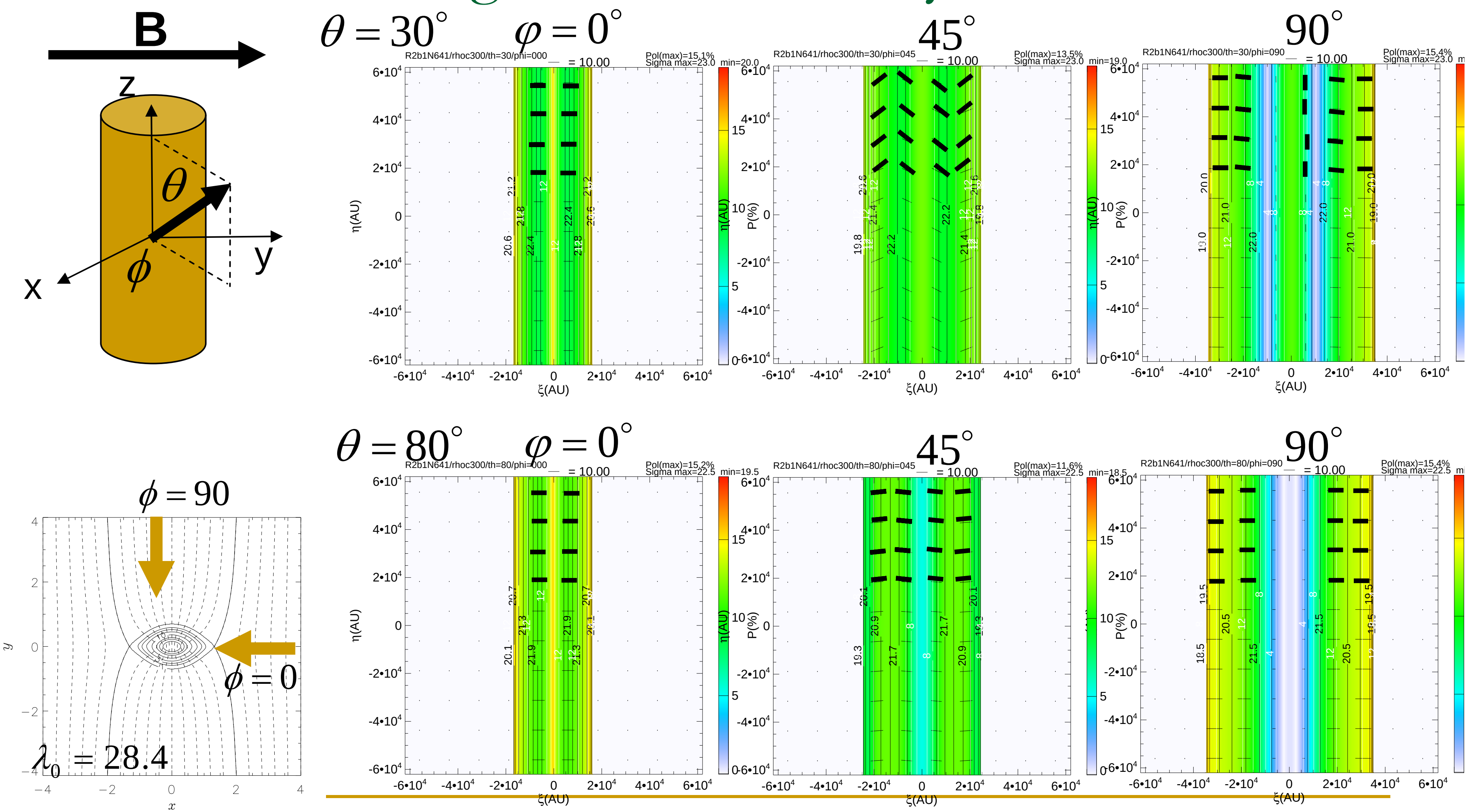


B ⊥ Filament → low pol. degree

* Pol.angle and degree depend on the direction of line of sight. ← B-field ~ uniform

Expected Polarization (Thermal Dust Emissions)

Models with High Central Density $\rho_c = 300\rho_s$



* Pol.angle and degree do not depend strongly on the direction of line of sight.

← B-field is squeezed near the equator.

Summary

- Structure of magnetohydrostatic filament is obtained.
  - Line-mass increases with the central density.
  - Max. line-mass supported by the magnetic flux is

$$\lambda_{\max} \approx 0.24\Phi_{cl,\,1\mathrm{D}} / G^{1/2} + 1.66 c_s^2 / G$$

  - There is a similarity between thin filament and disk.

$$M_{\max} \approx \Phi_{cl,\,2\mathrm{D}} / 2\pi G^{1/2}$$

- Expected Polarization (observational visualization)
  - low density contrast
    - From direction perp to global B-field → Pol. B-vector is observed perp to the filament.
    - From parallel to global B-field → Low polarization degree is expected.
  - high density contrast
    - Irrespective of the l.o.s. directions, pol. B-vector is observed perpendicular to the filament.
    - This is due to the squeezed B-field around the equator.
  - We can distinguish which configuration is realized in actual filaments.