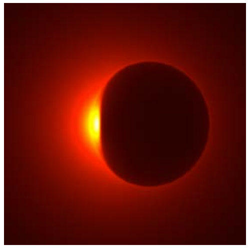

EANAM6, Kyung Hee University, Suwon, Korea
September 15-19, 2014

The Cutting-edge of Radiation Hydrodynamics

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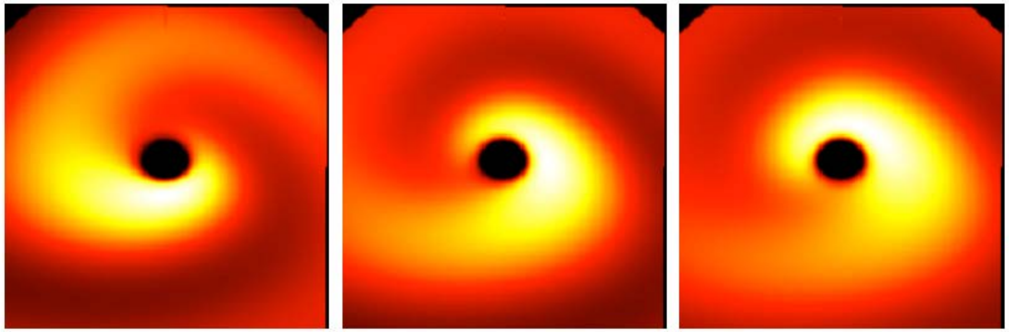
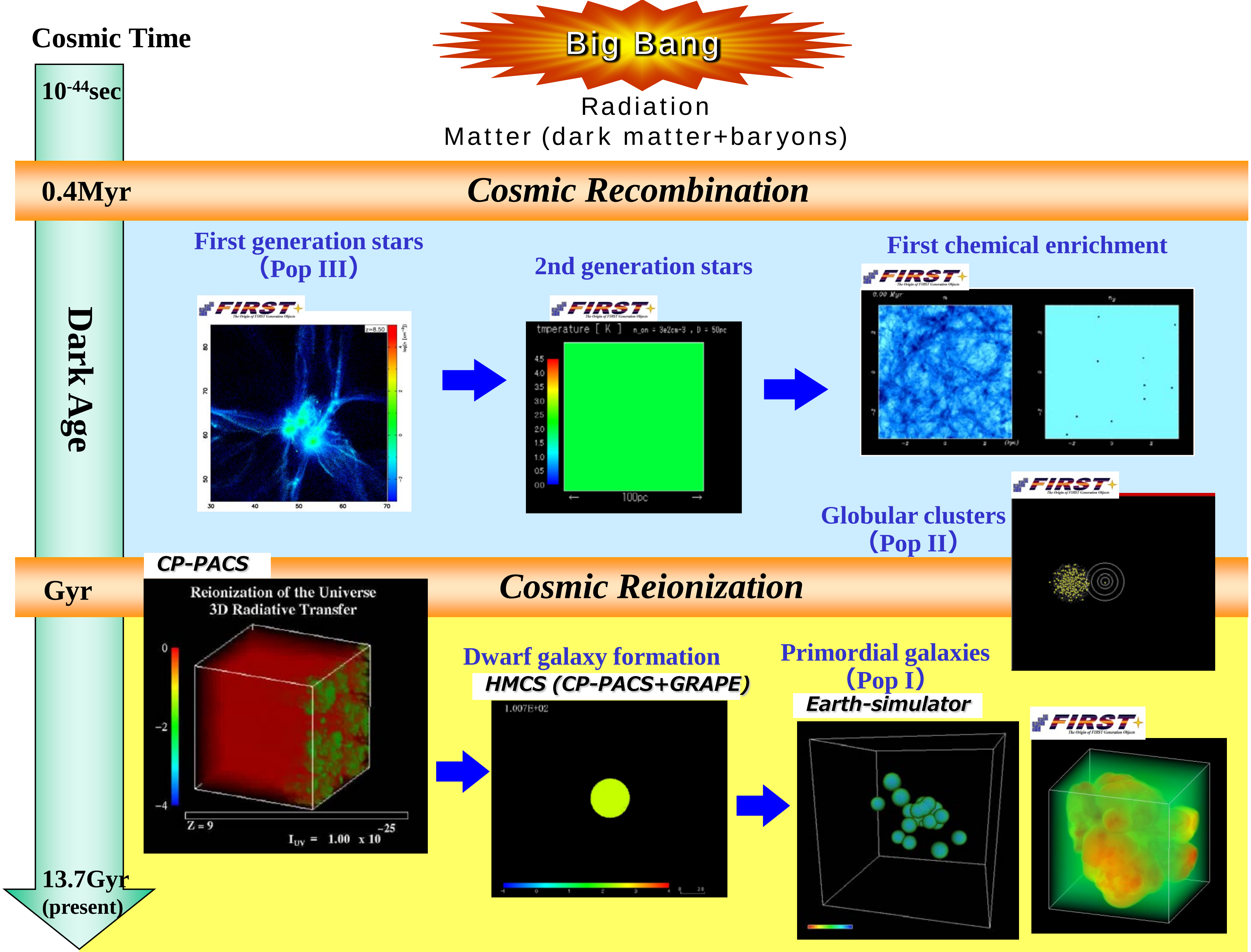
Outline

- Non-relativistic Radiation Transfer/Hydrodynamics
- Basic Equations of RHD and Closure Relations
- General Relativistic Radiation Transfer (GR-RT)
- Numerical Method of GR-RT
- Tests of GR-RT
- Summary

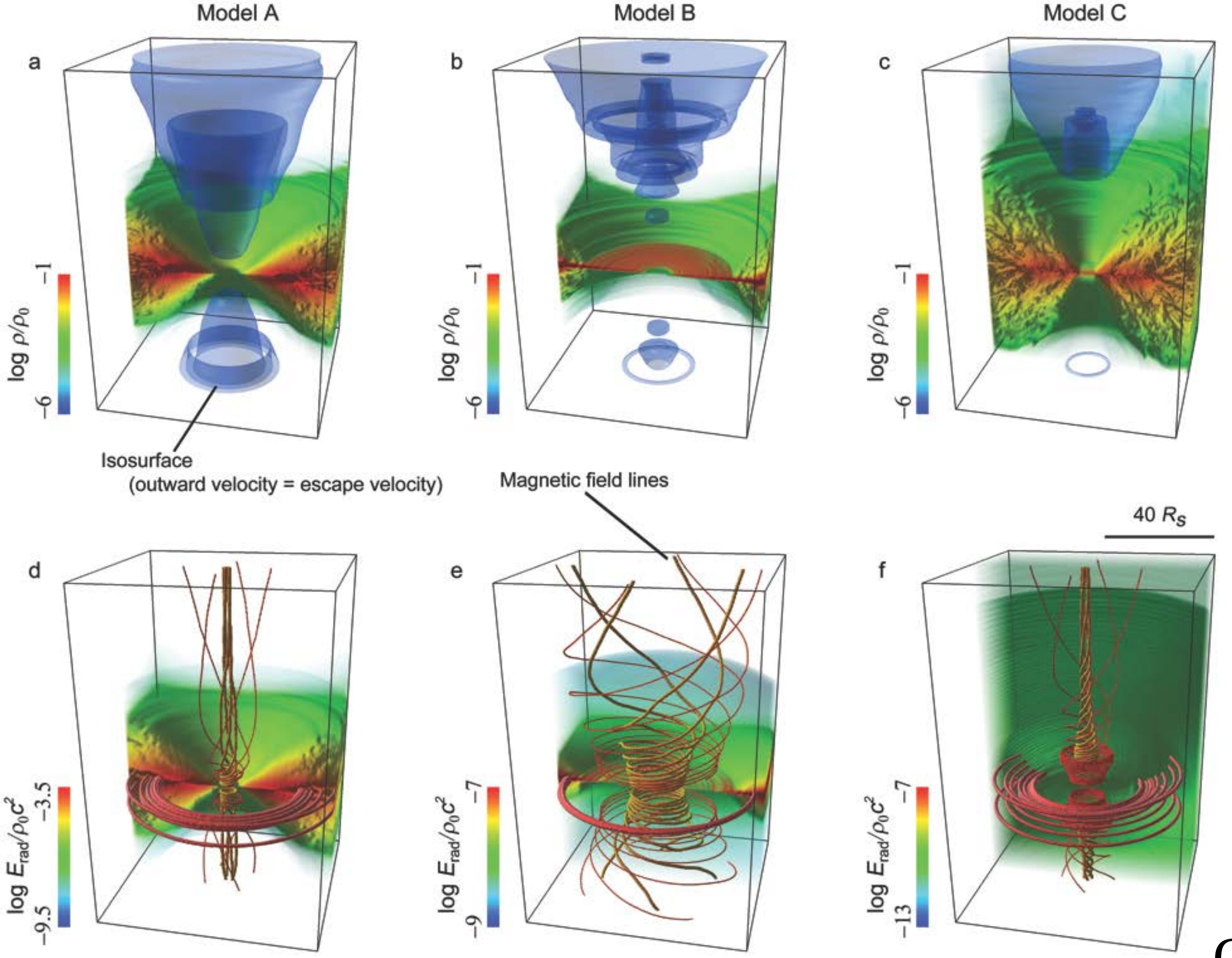



Cosmic Time
10^{-44}sec
Big Bang
Radiation
Matter (dark matter+baryons)
0.4Myr
Cosmic Recombination
Dark Age
First generation stars
(Pop III)
2nd generation stars
First chemical enrichment
FIRST
tmperature [K]
100pc
Globular clusters
(Pop II)
Gyr
CP-PACS
Reionization of the Universe
3D Radiative Transfer
Z = 9
Cosmic Reionization
Dwarf galaxy formation
HMCS (CP-PACS+GRAPE)
Primordial galaxies
(Pop I)
Earth-simulator
13.7Gyr
(present)

Radiation MHD Simulations on BH Accretion

Slim disk type	Standard-type	RIAF-type
$\dot{M} > \dot{M}_{Edd}$	$\dot{M} \sim \dot{M}_{Edd}$	$\dot{M} \ll \dot{M}_{Edd}$

FLD Approx.



Photon Trapping (GR effect)

Ohsuga+09

Conservation Law in RHD

$$(T^{\mu\nu} + R^{\mu\nu})_{;\nu} = F^{\mu}$$

Energy momentum tensor

$$T^{\mu\nu} = (\rho_0 + \rho_0 \varepsilon / c^2 + P / c^2) u^{\mu} u^{\nu} - P \eta^{\mu\nu}$$

$$R^{\mu\nu} = \begin{pmatrix} E & \mathbf{F}_i / c \\ \mathbf{F}_i / c & \mathbf{P}_{ij} \end{pmatrix}$$

I_ν: radiation energy density

$$E = \frac{1}{c} \int_0^{\infty} d\nu \int I_\nu d\Omega$$: radiation energy density

$$\mathbf{F}_i = \int_0^{\infty} d\nu \int I_\nu \mathbf{n}_i d\Omega$$: radiation flux

$$\mathbf{P}_{ij} = \frac{1}{c} \int_0^{\infty} d\nu \int I_\nu \mathbf{n}_i \mathbf{n}_j d\Omega$$: radiation stress tensor

Moment Equations & Closure Relation

Energy Equation **(1 equation)**

$$\frac{p}{\Gamma-1}\frac{d}{dt}\ln\left(\frac{T}{\rho^{\Gamma-1}}\right)=-\left(\frac{\partial E}{\partial t}+\nabla\cdot\mathbf{F}\right)+\mathbf{v}\cdot\left(\frac{1}{c^2}\frac{\partial\mathbf{F}}{\partial t}+\nabla\cdot\mathbf{P}\right)$$

Moment Equations **(4 equations)**

$$\frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}}+\nabla\cdot\mathbf{F}=\int_0^\infty d\nu\int\chi_\nu(S_\nu-I_\nu)d\Omega$$

$$\frac{1}{c^2}\frac{\partial\mathbf{F}}{\partial \boldsymbol{t}}+\nabla\cdot\mathbf{P}=\frac{1}{c}\int_0^\infty d\nu\int\chi_\nu(S_\nu-I_\nu)\mathbf{n}\,d\Omega$$

In total, 5 equations

Ten variables: *T*, *E*, F(3 components), P(6 components)

Closure relation is required !

Closure Relations

FLD (Flux Limited Diffusion)

information of E

difficulty: aspherical fields

M1 Closure

information of E, F

difficulty: collision of wave fronts

3D problem

VET (Variable Eddington Tensor)

$f_{ij} \equiv P_{ij}/E$

information of E, F, P

difficulty: high dimensionality

6D problem

General Relativistic Simulations

- **GR MHD simulation [no radiation]**

  Koide + 1999, Hawley + 2000, Gammie + 2003,

  Komissarov 2005, Duez+2005, Shibata & Sekiguchi 2005,

  Nagataki 2009 etc.

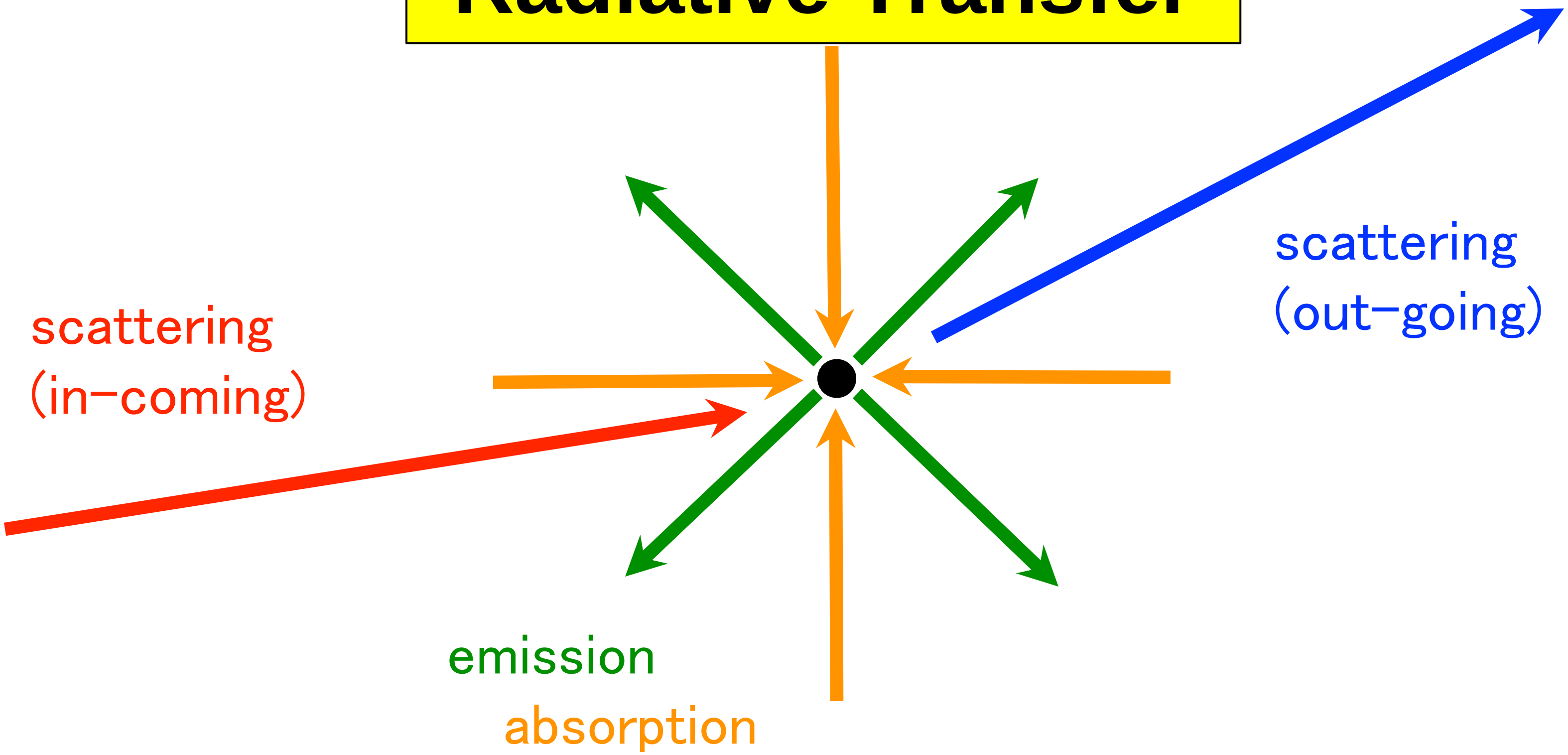
- **GR radiation MHD simulation [no radiative transfer]**

  Farris+ 2008 (FLD), Zanotti+2011(FLD),

  Shibata+ 2012 (M1 closure), Fragile+2012(FLD),

  Sadowski+ 2012 (M1 closure) etc.

Radiative Transfer



Time-dependent Radiative Transfer Equation

(Photon Boltzmann equation in phase space of 3D space, 2D direction, and 1D frequency.)

$$\frac{1}{c}\frac{\partial I_\nu(\mathbf{n})}{\partial t} + \mathbf{n}\cdot\nabla I_\nu(\mathbf{n}) = \frac{\eta_\nu}{4\pi} - \kappa_\nu I_\nu(\mathbf{n}) - \sigma_\nu I_\nu(\mathbf{n}) + \sigma_\nu \int \phi(\mathbf{n};\mathbf{n}') I_\nu(\mathbf{n}') d\Omega'$$

emission

absorption

scattering (out-going)

scattering (in-coming)

GR Radiation Transfer

General Relativistic Boltzmann Equation of Photons

$$\frac{d\mathcal{I}_\nu}{d\lambda} = \mathcal{E}_\nu - A_\nu \mathcal{I}_\nu$$

$\mathcal{I}_\nu \equiv \frac{I_\nu}{\nu^3}$: Invariant specific intensity

$\mathcal{E}_\nu \equiv \frac{\eta_\nu}{\nu^2}$: Invariant emissivity

$A_\nu \equiv \nu\chi_\nu$: Invariant extinction

- **Solve GR radiative transfer along geodesics**
- **Obtain invariant specific intensity in 6D phase space**

Difficulties in GR-RHD

A) In relativistic motion, the steady state of radiation fields cannot be assumed.

[Time-dependent transfer equation should be solved.]

C) Light bending, frame-dragging, and gravitational redshifts should be included.

[Transfer should be solved along the geodesics.]

B) Causality should be retained.

[We should solve the propagation of wave fronts in proper time.]

D) GR energy-momentum tensor of radiation should be obtained.

[LNRF (locally non-rotating reference frame) should transformed to the curved space.]

We have overcome all these difficulties !

General Relativistic Radiative Transfer

① Non-relativistic RT
(ART/Long Characteristic)

* absorption/scattering
* coupled with hydrodynamics
* parallelization

② GR Ray-Tracing

* special/general relativistic effects
* 6D phase space
* no absorption/scattering

merged

Time-dependent GR radiative transfer solver

* Photon Boltzmann equation in 6D phase space is solved
* Emission, absorption, scattering are included, consistently with special/general relativistic effects.
* Parallelization is achieved with GPU.

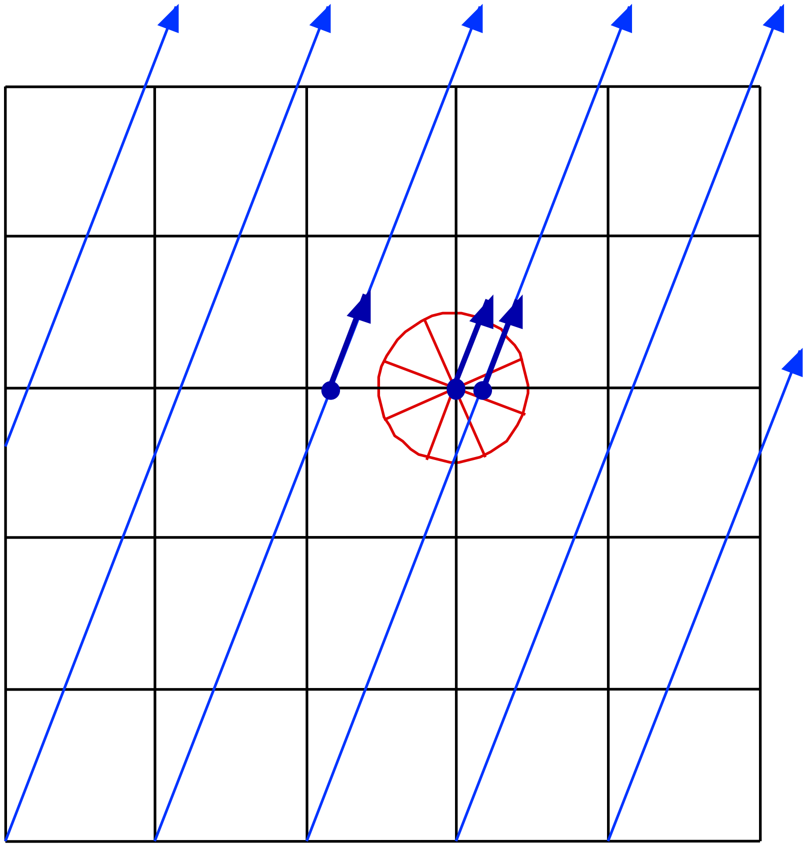
ART method

(Authentic Radiative Transfer)

Non-relativistic Steady Transfer

$$\mathbf{n}\cdot\nabla I_\nu(\mathbf{n}) = -\kappa_\nu I_\nu(\mathbf{n}) + \eta_\nu / 4\pi - \sigma_\nu I_\nu(\mathbf{n}) + \sigma_\nu \int \phi(\mathbf{n};\mathbf{n}') I_\nu(\mathbf{n}') d\Omega'$$

Transfer is solved along a long ray across the domain



- Physical quantities are interpolated at each grid
- A bit complex coding
- No numerical diffusion (accuracy equivalent to long char.)
- Operations (same as short char.)

$$\sim \quad N_x N_y N_z \cdot N_\theta N_\varphi N_\nu$$

Calculations of Radiation Tensor

specific intensities $\mathcal{I}$ ●

(in the phase space)

⬇ integration in the momentum space

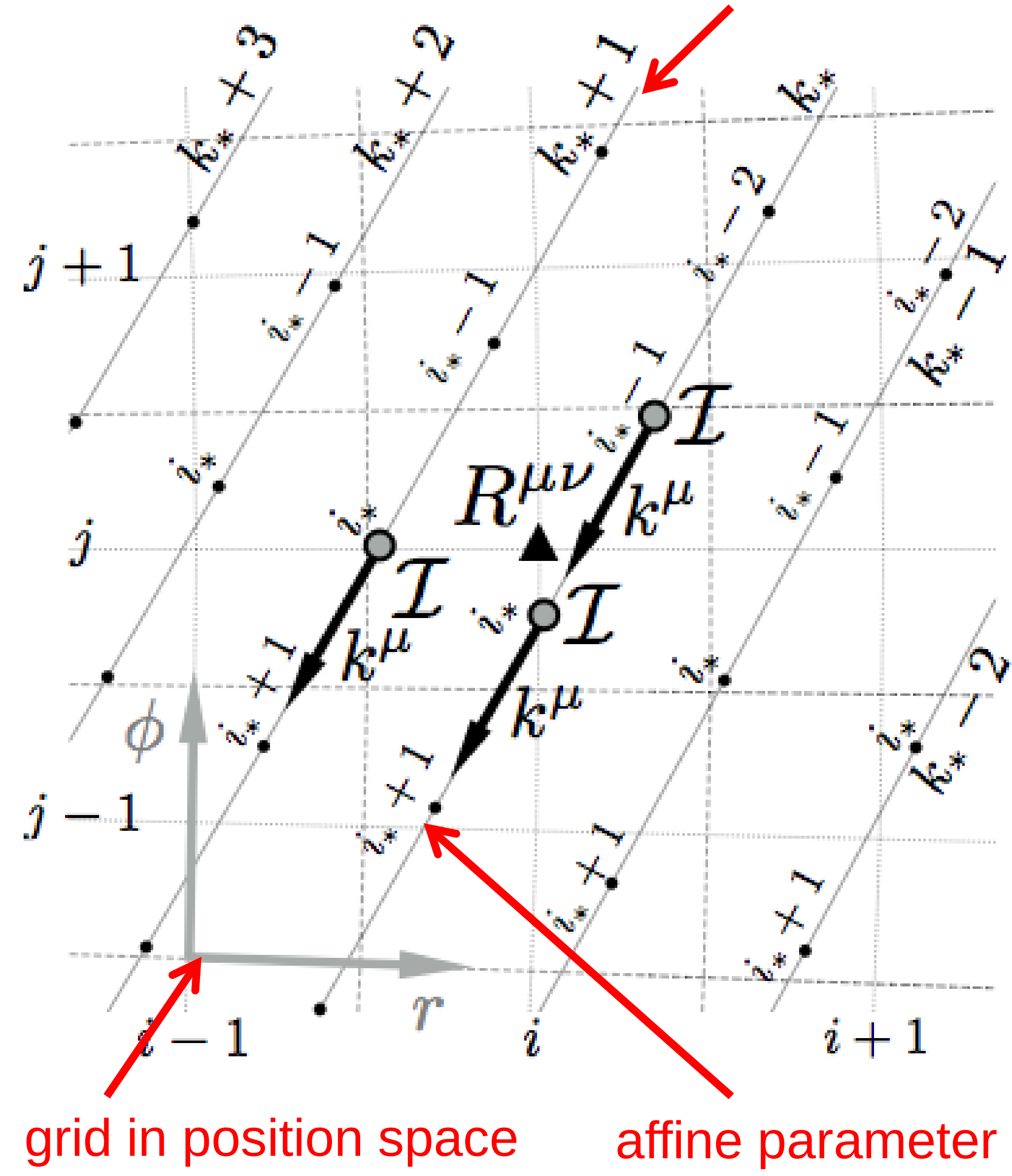
radiation tensor $R^{\mu\nu}$ ▲

in the nearest neighbors of $\mathcal{I}$

(in the position space)

$$R^{\mu\nu} = \int k^\mu k^\nu \mathcal{I} \, dP$$

➡ Eddington tensor f_{ij}



Tetrad formalism

Mix-frame approach

LNRF (locally non-rotating reference frame)
(Local Minkovski spacetime)

Global curved spacetime

$$(T^{\mu\nu} + R^{\mu\nu})_{,\nu} = F^{\mu} \quad \Leftrightarrow \quad (T^{\alpha\beta} + R^{\alpha\beta})_{;\beta} = F^{\alpha}$$

Conservation Law of RHD

$$R^{\alpha\beta} = \varepsilon^{\alpha}_{\mu}\varepsilon^{\beta}_{\nu} R^{\mu\nu}$$

$$T^{\mu\nu} = (\rho_0 + \rho_0\varepsilon/c^2 + P/c^2)u^{\mu}u^{\nu} - P\eta^{\mu\nu}$$

$$R^{\mu\nu} = \begin{pmatrix} E & \mathbf{F}/c \\ \mathbf{F}/c & \mathbf{P} \end{pmatrix}$$

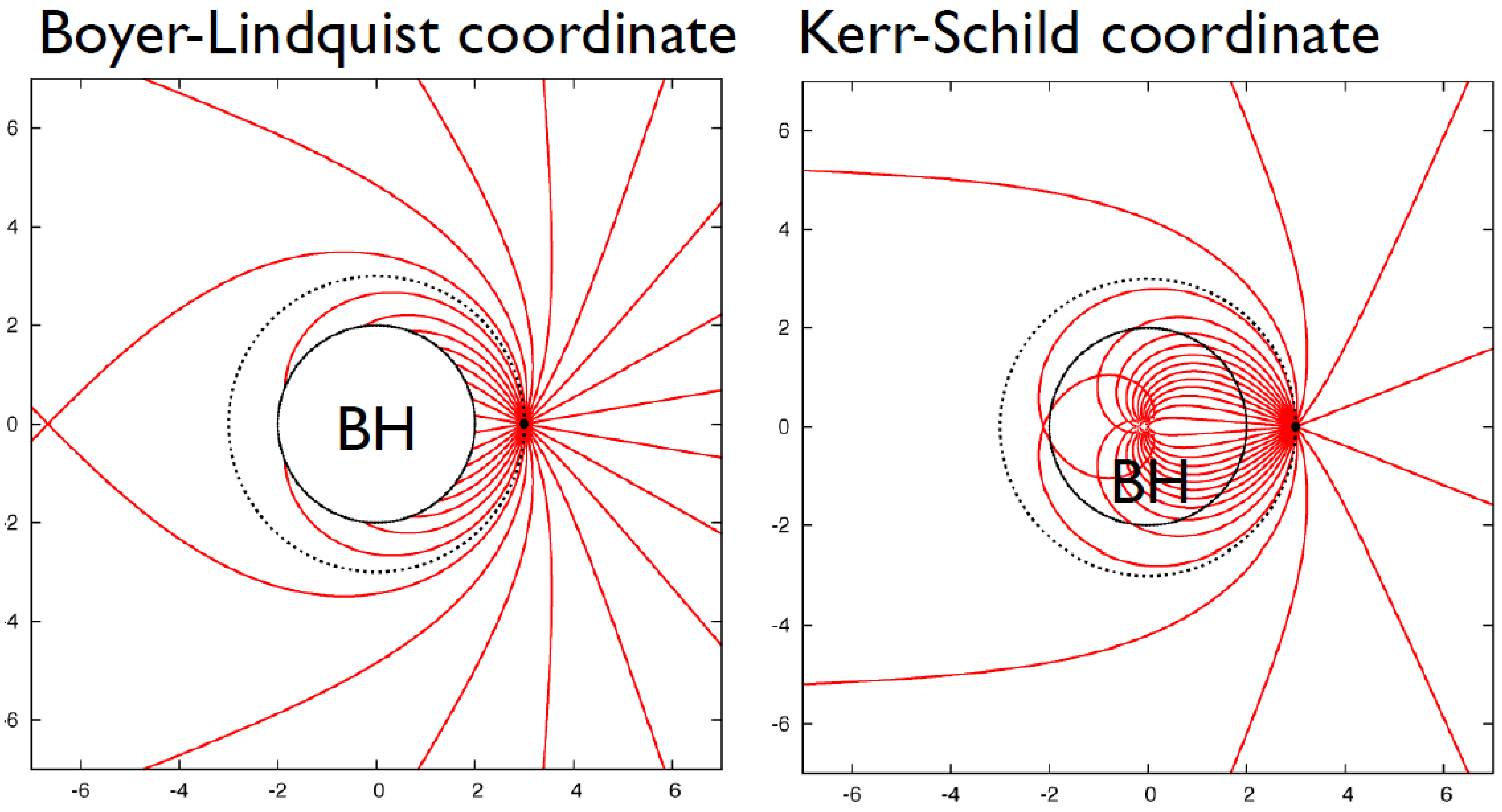
$$E = \frac{1}{c}\int_0^{\infty} d\nu \int I_{\nu} d\Omega \text{ : radiation energy density}$$

$$\mathbf{F} = \int_0^{\infty} d\nu \int I_{\nu} \mathbf{n} d\Omega \text{ : radiation flux}$$

$$\mathbf{P} = \frac{1}{c}\int_0^{\infty} d\nu \int I_{\nu} \mathbf{nn} d\Omega \text{ : radiation stress tensor}$$

Coordinates in Curved Space

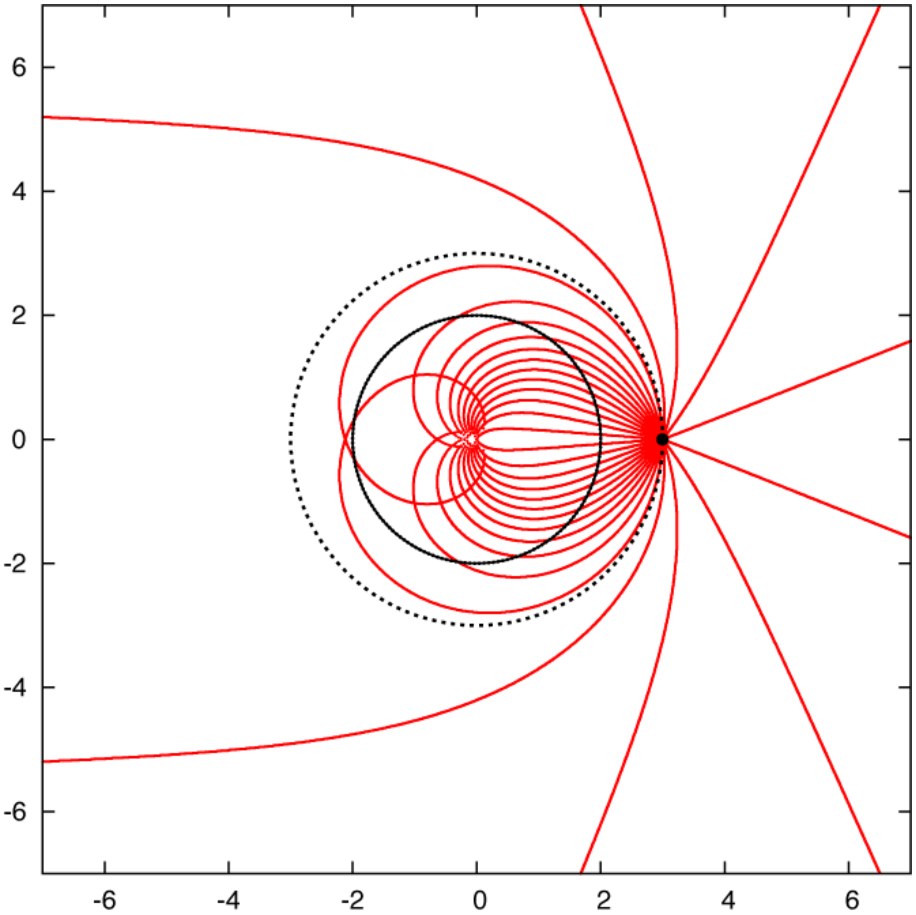
Horizon capture

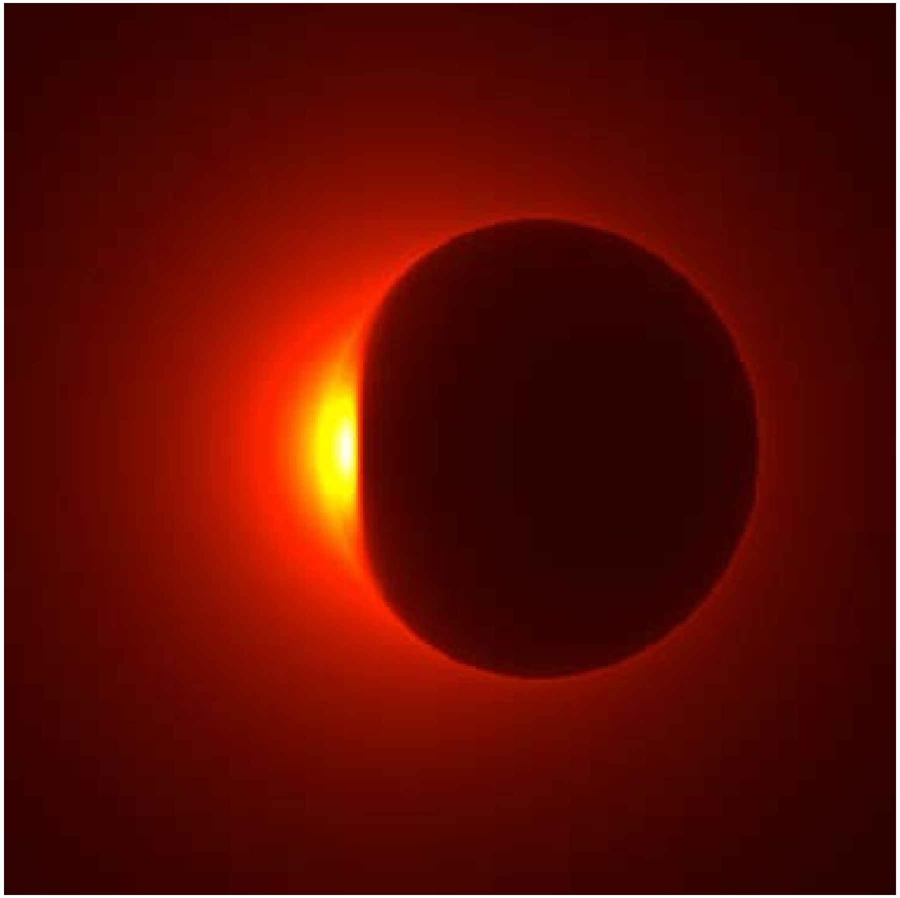


Static test: BH shadow

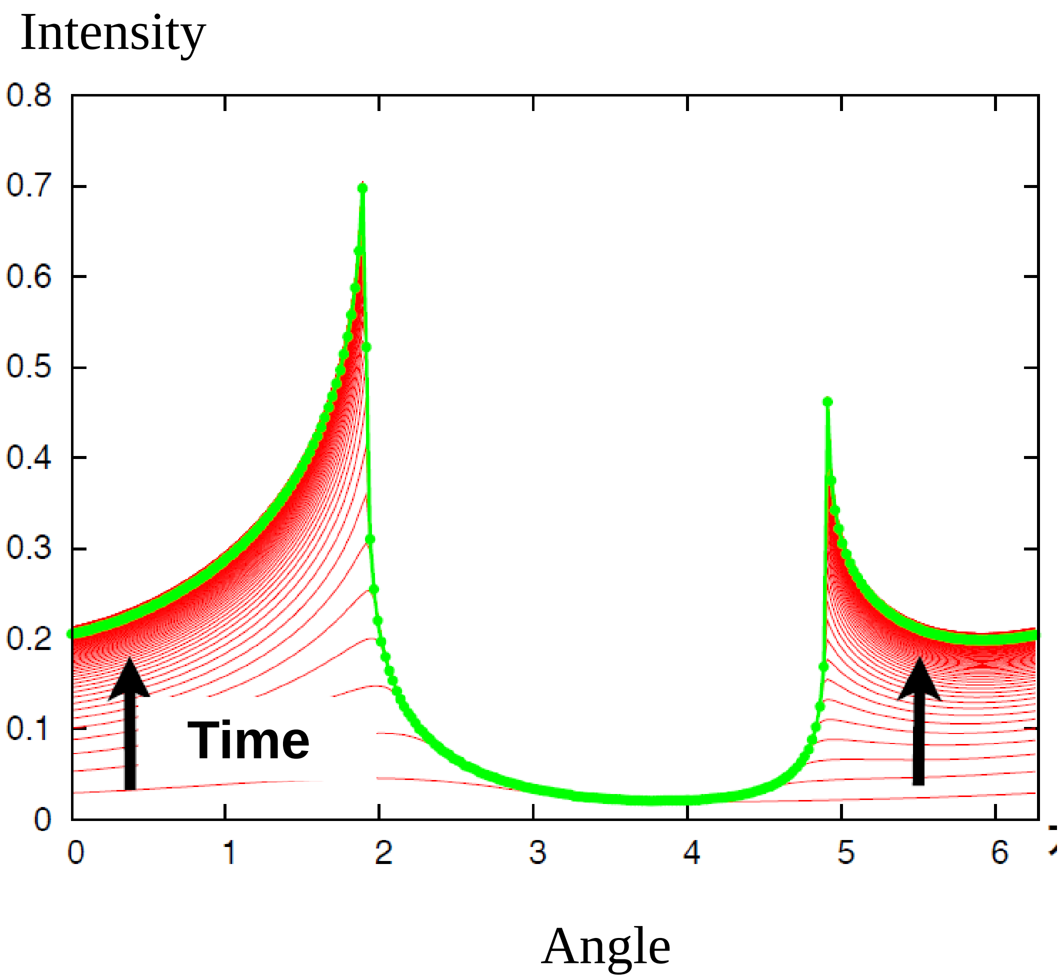
Takahashi & Umemura 2014, in prep

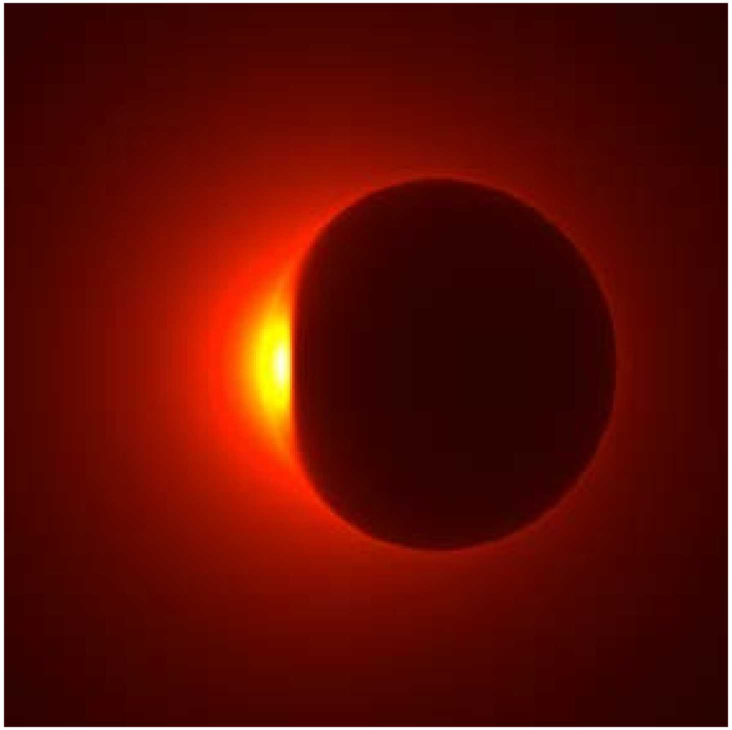
A source is located behind a BH.
A shadow forms by light bending + frame-dragging + gravitational redshift.





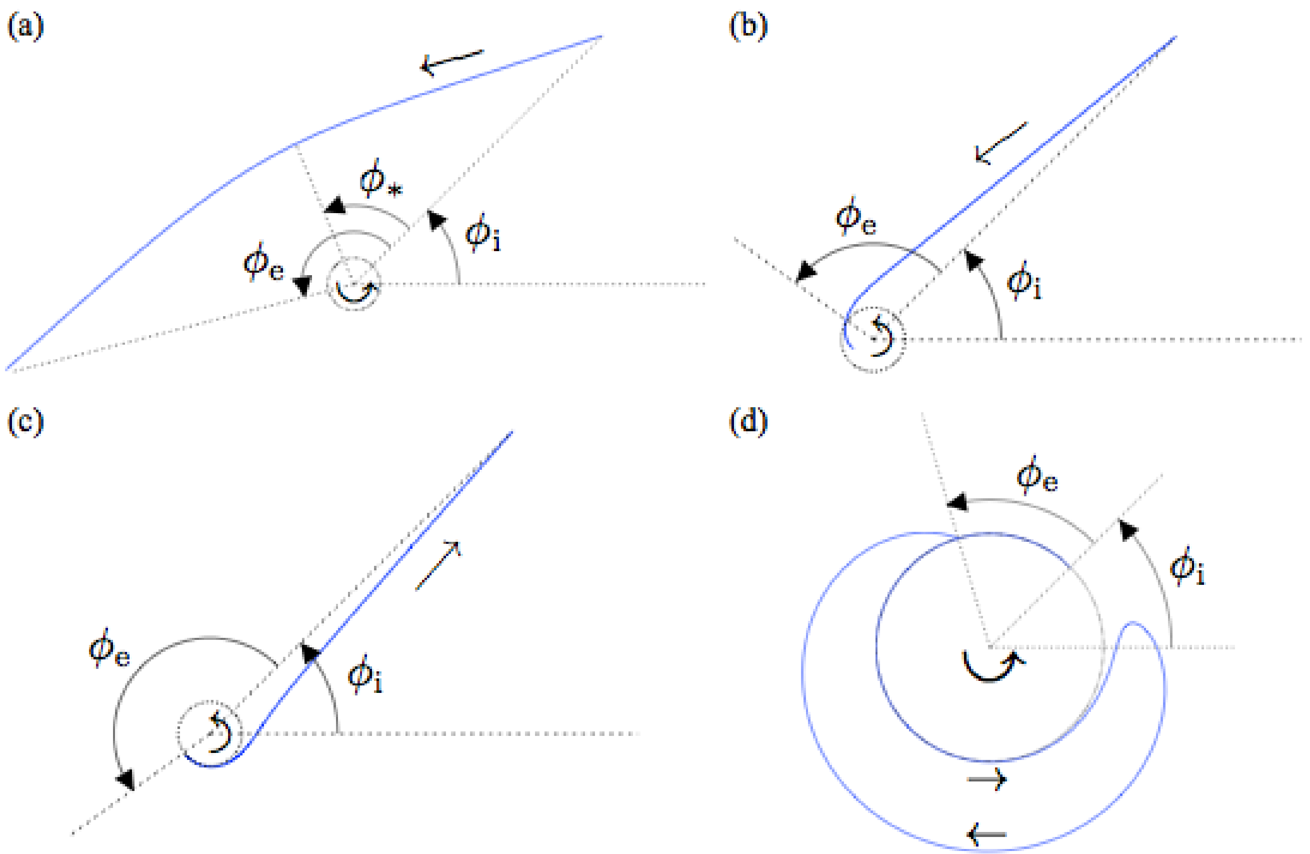
Time Evolution of Invariant Brightness





Capture of Wave Front

Geodesics Patterns and Grids in Phase Space



A. angular coordinates at initial points

$$(\theta_\mathrm{i}, \phi_\mathrm{i})$$

B. angular coordinates at end points

$$(\theta_\mathrm{e}, \phi_\mathrm{e})$$

C. angular coordinates in LNRF at initial points

$$(\bar{\theta}, \bar{\phi})$$

(direction of geodesics)

D. angular coordinates measured from $(\theta_\mathrm{i}, \phi_\mathrm{i})$

$$(\theta_*, \phi_*)$$

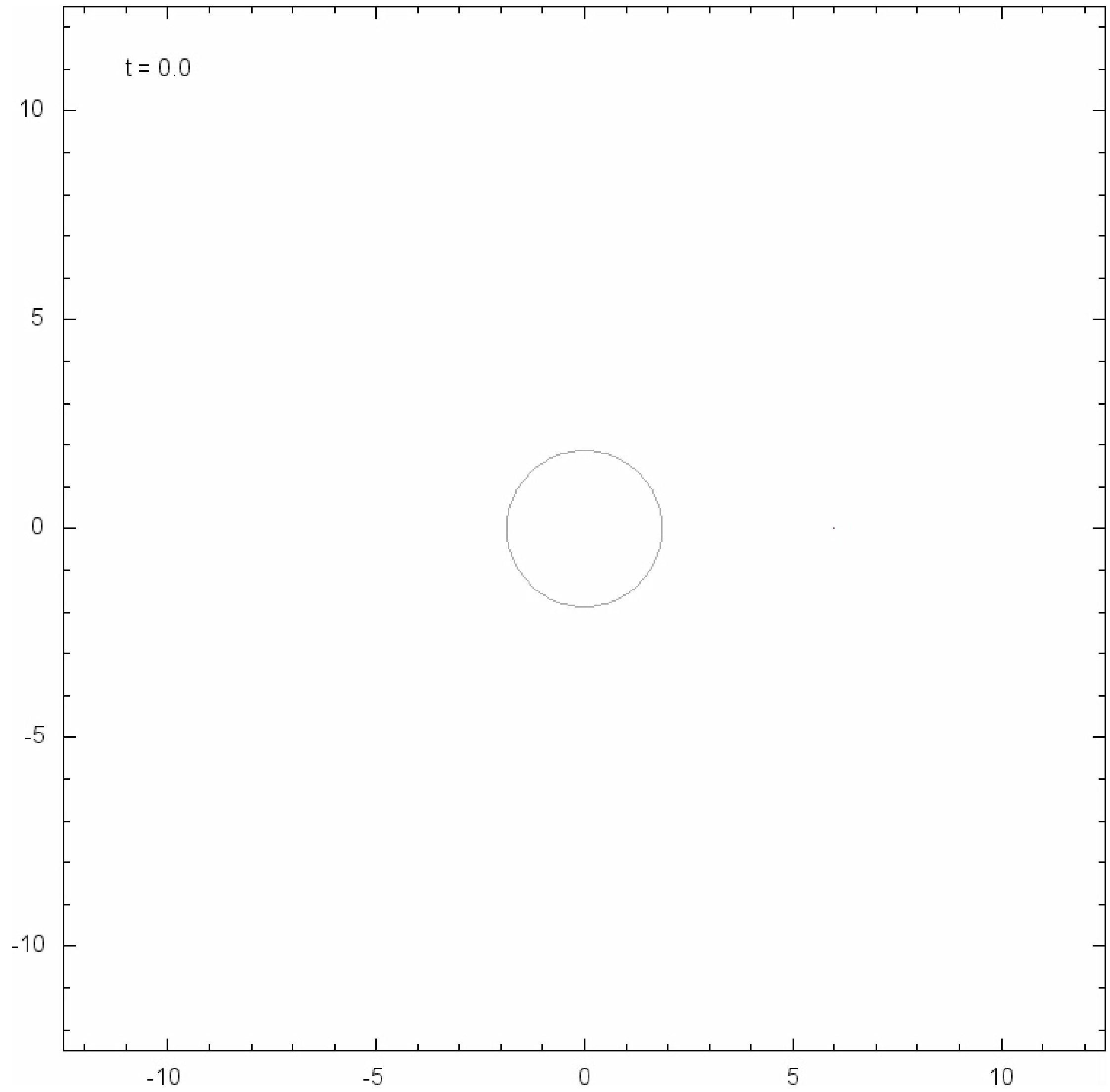
Geodesics are calculated for $\left\{ \begin{array}{l} 0 \le \theta_i \le \pi, \ 0 \le \phi_i \le 2\pi \\ -3\pi \le \phi_\mathrm{e} \le 3\pi \ \text{ for } \ 0 \le \bar{\phi} \le 2\pi \end{array} \right.$

1. all possible patterns of geodesics in phase space.

2. relativistic orbits with $-3\pi \le \phi_\mathrm{e} \le 3\pi$ **are included.**

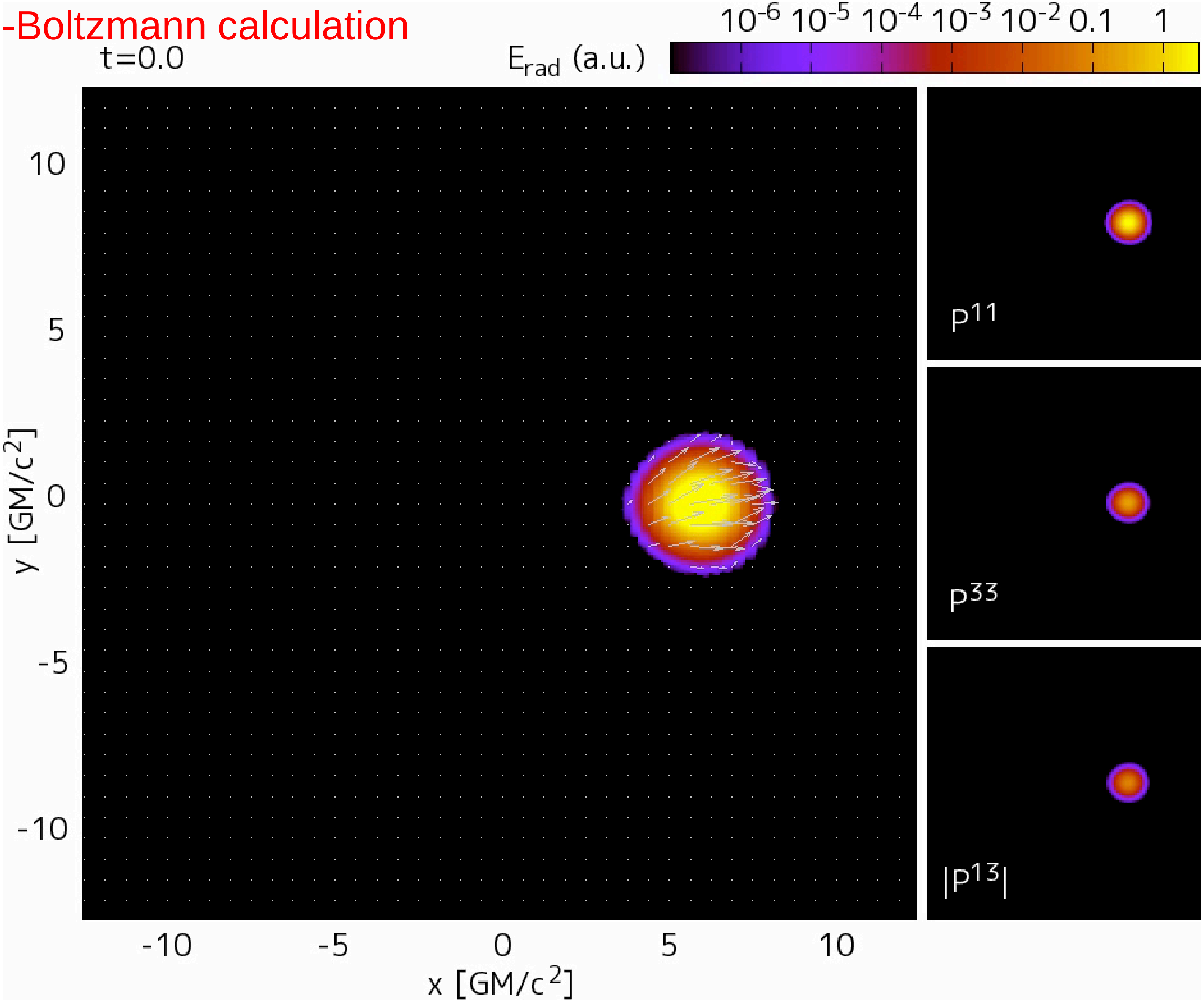
Ray-tracing calculation

Light ray rotates
5 times around a BH



Dynamical Test 1: Wave-front propagation

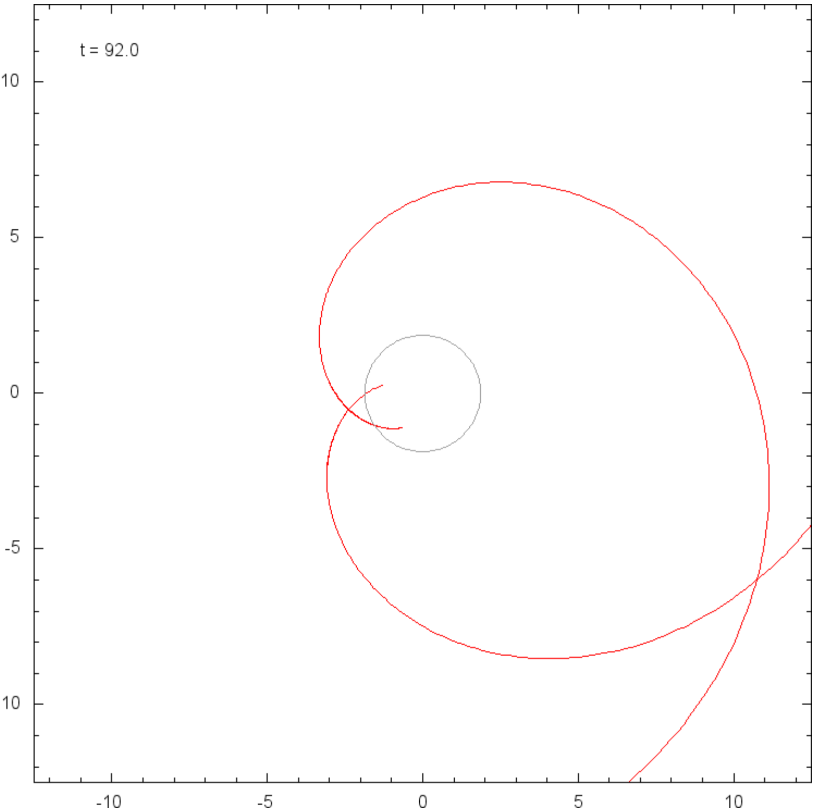
GR-Boltzmann calculation



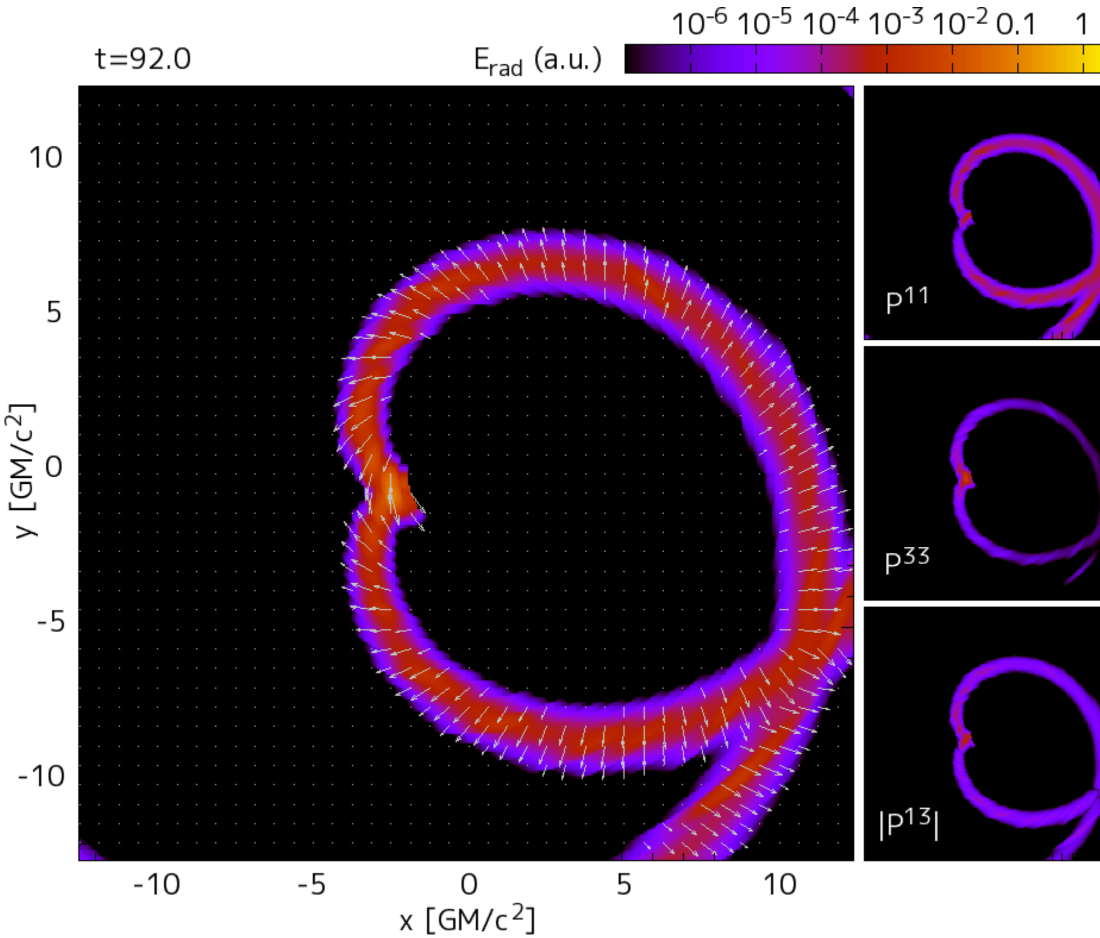
Comparison: Wave-front propagation

Takahashi & Umemura 2014, in prep

Ray-tracing



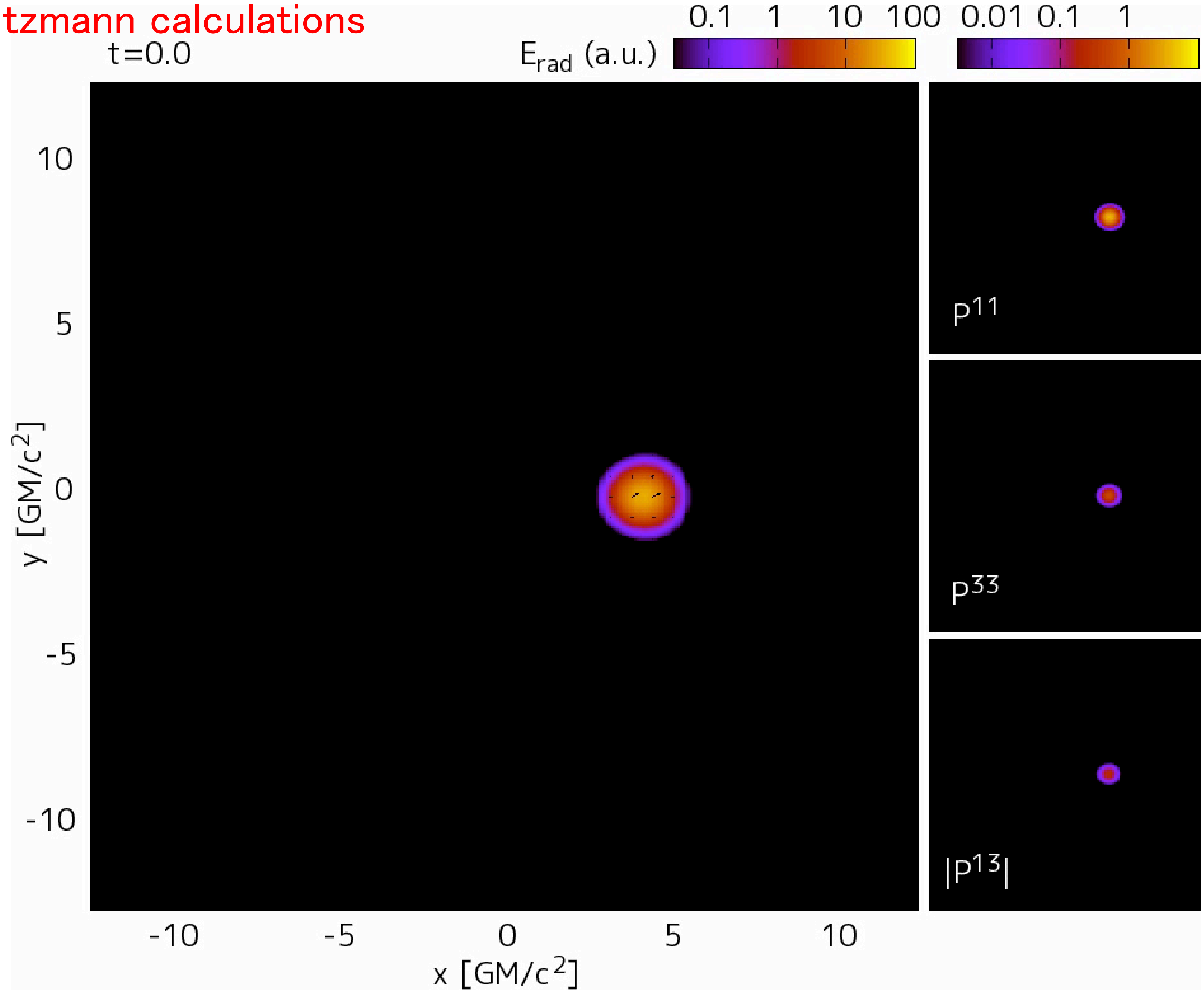
Boltzmann

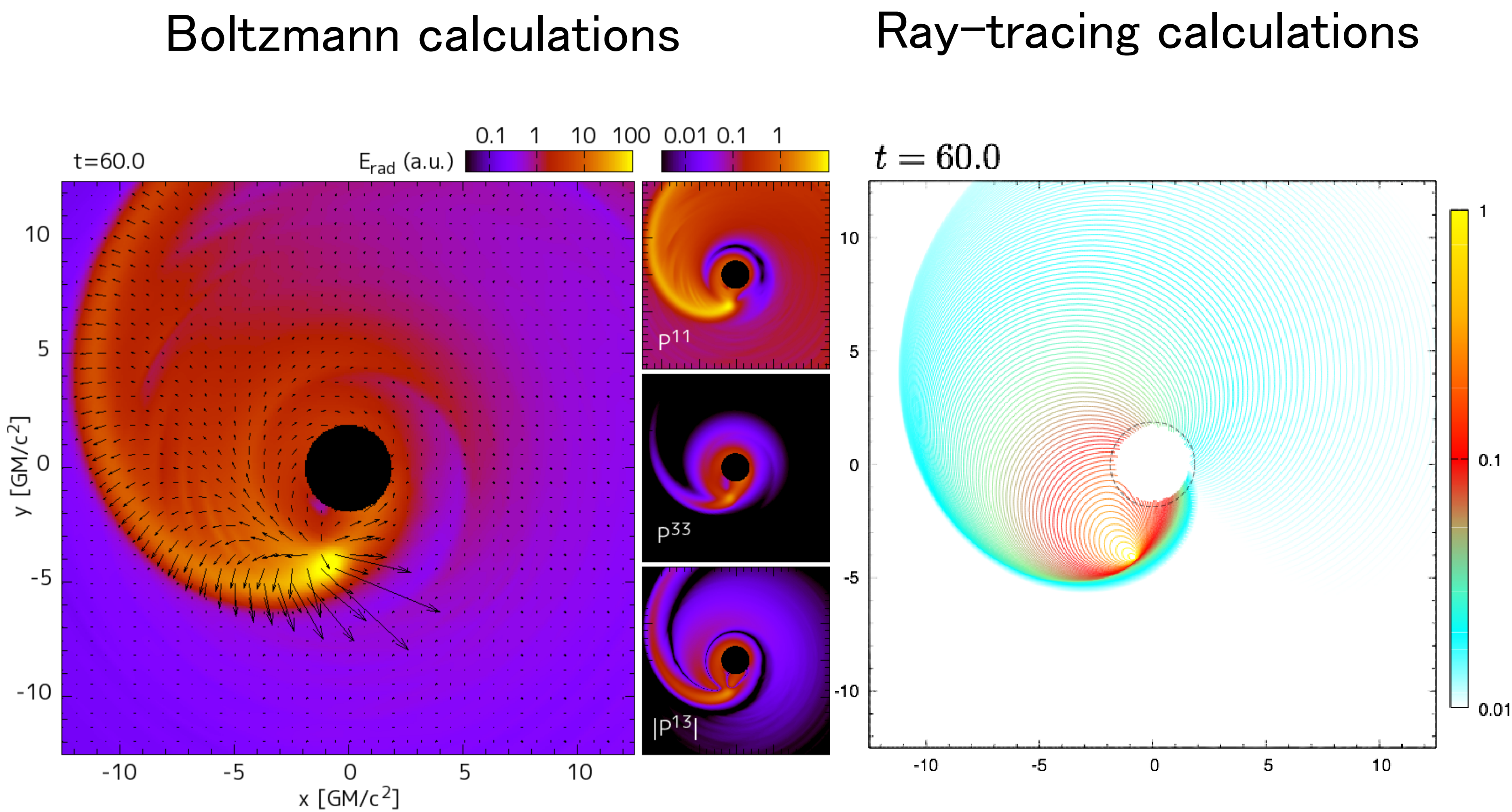


Wave fronts can cross each other without any collision !

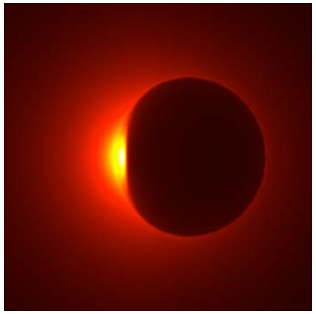
Dynamical Test 2: Photon wave front from a rotating hot spot

Boltzmann calculations





N_i (r)=90, N_j (ϕ) =256
Geodesics=4608

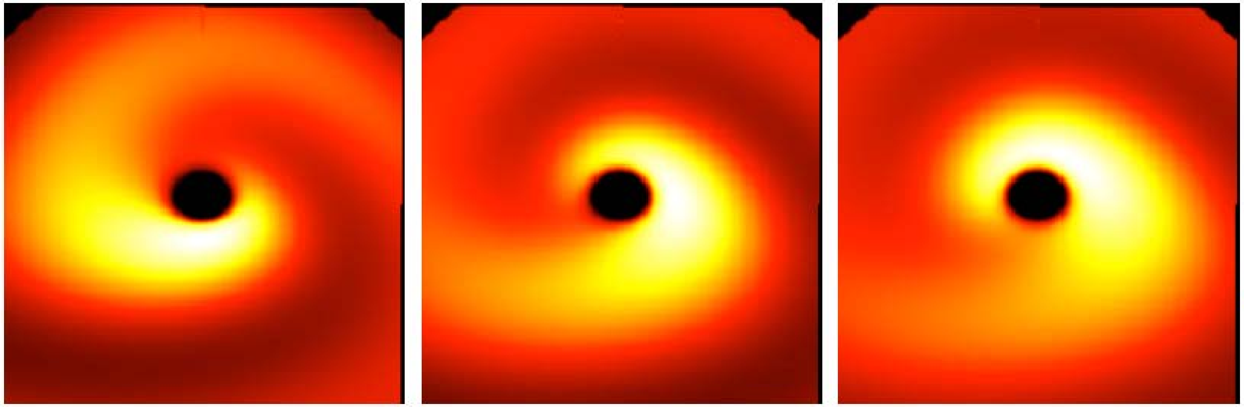


Vermeer

Master of light

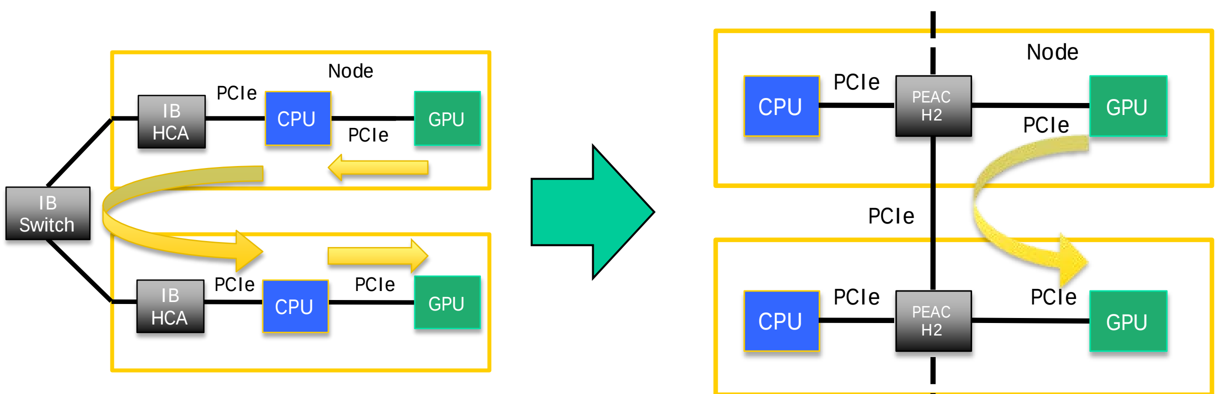
Variable Eddington-tensor Radation-hydrodynamics with Metric Enchained Ray-tracing

General Relativistic Radiation Transfer
Radiation Hydrodynamics in Curved Space



HA-PACS (PACS-VIII) system

- System spec.
  - 268 + 64 (TCA) nodes
  - CPU 89+28.7TFLOPS
    + GPU 713T+335.4FLOPS
    =802+364TFLOPS
    = total 1.166PFLOPS
  - Memory 34TByte、memory bandwidth 26TByte/sec
  - Bi-section bandwidth 2.1TByte/s
  - Storage 504TByte
  - Power 408kW



True GPU-direct
With cooperation of NVIDA

Summary

- **GR radiative transfer solver "*Vermeer*"**

The time-dependent Boltzmann equation of photons is directly integrated along the geodesics. The propagation of wave fronts is solved in proper time, so that the causality is completely retained.

- **Tests in a Kerr metric**

(1) BH shadow
(2) Wave front propagation
(3) Propagation of radiation wave from a rotating hot spot

- **Moment calculations**

Eddington tensor of radiation in LNRF (locally non-rotating reference frame) is transformed to that in the curved space, which can close GR-RHD equations properly.

- **Parallel calculations**

The scheme can be parallelized with GPUs.