The Cutting-edge of Radiation Hydrodynamics

Masayuki Umemura
Director
Center for Computational Sciences, University of Tsukuba

Collaborator
Rota Takahashi
Tomakomai National Collage of Technology
Outline

- Non-relativistic Radiation Transfer/Hydrodynamics
- Basic Equations of RHD and Closure Relations
- General Relativistic Radiation Transfer (GR-RT)
- Numerical Method of GR-RT
- Tests of GR-RT
- Summary
Cosmic Time

- $10^{-44}\text{sec}$
- 0.4 Myr
- 13.7 Gyr (present)

Cosmic Recombination

- First generation stars (Pop III)
- 2nd generation stars
- First chemical enrichment

Globular clusters (Pop II)

Cosmic Reionization

- Dwarf galaxy formation
- Primordial galaxies (Pop I)
- Earth-simulator
Radiation MHD Simulations on BH Accretion

Slim disk type
\[ \dot{M} > \dot{M}_{Edd} \]
Model A

Standard-type
\[ \dot{M} \sim \dot{M}_{Edd} \]
Model B

RIAF-type
\[ \dot{M} \ll \dot{M}_{Edd} \]
Model C

Isosurface (outward velocity = escape velocity)

Magnetic field lines

Photon Trapping (GR effect)

FLD Approx.

Ohsuga+09
Conservation Law in RHD

\[(T^{\mu\nu} + R^{\mu\nu})_{;\nu} = F^\mu\]

Energy momentum tensor

\[
T^{\mu\nu} = (\rho_0 + \rho_0 \epsilon/c^2 + P/c^2) u^\mu u^\nu - P \eta^{\mu\nu}
\]

\[
R^{\mu\nu} = \begin{pmatrix}
E & F_i / c \\
F_i / c & P_{ij}
\end{pmatrix}
\]

\(I_\nu\): radiation energy density

\[
E = \frac{1}{c} \int_0^\infty dv \int I_\nu d\Omega : \text{radiation energy density}
\]

\[
F_i = \int_0^\infty dv \int I_\nu n_i d\Omega : \text{radiation flux}
\]

\[
P_{ij} = \frac{1}{c} \int_0^\infty dv \int I_\nu n_i n_j d\Omega : \text{radiation stress tensor}
\]
Moment Equations & Closure Relation

Energy Equation (1 equation)
\[
\frac{p}{\Gamma - 1} \frac{dT}{dt} \ln \left( \frac{T}{\rho^{\Gamma - 1}} \right) = - \left( \frac{\partial E}{\partial t} + \nabla \cdot F \right) + \mathbf{v} \cdot \left( \frac{1}{c^2} \frac{\partial F}{\partial t} + \nabla \cdot \mathbf{P} \right)
\]

Moment Equations (4 equations)
\[
\frac{\partial E}{\partial t} + \nabla \cdot F = \int_0^\infty dv \int \chi_v (S_v - I_v) d\Omega
\]
\[
\frac{1}{c^2} \frac{\partial F}{\partial t} + \nabla \cdot \mathbf{P} = \frac{1}{c} \int_0^\infty dv \int \chi_v (S_v - I_v) n d\Omega
\]

In total, 5 equations

Ten variables: \( T, E, F(3 \text{ components}), P(6 \text{ components}) \)

Closure relation is required!
Closure Relations

**FLD (Flux Limited Diffusion)**
- information of $E$
- difficulty: aspherical fields

**M1 Closure**
- information of $E$, $F$
- difficulty: collision of wave fronts

**VET (Variable Eddington Tensor)**
\[ f_{ij} = \frac{P_{ij}}{E} \]
- information of $E$, $F$, $P$
- difficulty: high dimensionality

3D problem

6D problem
General Relativistic Simulations

- **GR MHD simulation [no radiation]**
  - Koide + 1999, Hawley + 2000, Gammie + 2003,
  - Komissarov 2005, Duez+2005, Shibata & Sekiguchi 2005,
  - Nagataki 2009 etc.

- **GR radiation MHD simulation [no radiative transfer]**
  - Farris+ 2008 (FLD), Zanotti+2011(FLD),
  - Shibata+ 2012 (M1 closure), Fragile+2012(FLD),
  - Sadowski+ 2012 (M1 closure) etc.
Time-dependent Radiative Transfer Equation

(Photon Boltzmann equation in phase space of 3D space, 2D direction, and 1D frequency.)

\[
\frac{1}{c} \frac{\partial I_\nu(n)}{\partial t} + n \cdot \nabla I_\nu(n) = \frac{\eta_\nu}{4\pi} - \kappa_\nu I_\nu(n) - \sigma_\nu I_\nu(n) + \sigma_\nu \int \phi(n; n') I_\nu(n') \, d\Omega'
\]
GR Radiation Transfer

General Relativistic Boltzmann Equation of Photons

\[
\frac{d \mathcal{S}_\nu}{d \lambda} = \mathcal{E}_\nu - A_\nu \mathcal{S}_\nu
\]

\(\mathcal{S}_\nu \equiv \frac{I_\nu}{\nu^3} \): Invariant specific intensity

\(\mathcal{E}_\nu \equiv \frac{\eta_\nu}{\nu^2} \): Invariant emissivity

\(A_\nu \equiv \nu \chi_\nu \): Invariant extinction

➢ Solve GR radiative transfer along geodesics
➢ Obtain invariant specific intensity in 6D phase space
Difficulties in GR-RHD

A) In relativistic motion, the steady state of radiation fields cannot be assumed.
   [Time-dependent transfer equation should be solved.]

C) Light bending, frame-dragging, and gravitational redshifts should be included.
   [Transfer should be solved along the geodesics.]

B) Causality should be retained.
   [We should solve the propagation of wave fronts in proper time.]

D) GR energy-momentum tensor of radiation should be obtained.
   [LNRF (locally non-rotating reference frame) should be transformed to the curved space.]  

We have overcome all these difficulties!
General Relativistic Radiative Transfer

① Non-relativistic RT (ART/Long Characteristic)
  * absorption/scattering
  * coupled with hydrodynamics
  * parallelization

② GR Ray-Tracing
  * special/general relativistic effects
  * 6D phase space
  * no absorption/scattering

merged

Time-dependent GR radiative transfer solver
  * Photon Boltzmann equation in 6D phase space is solved
  * Emission, absorption, scattering are included, consistently with special/general relativistic effects.
  * Parallelization is achieved with GPU.
Non-relativistic Steady Transfer

\[ \mathbf{n} \cdot \nabla I_\nu (\mathbf{n}) = -\kappa_\nu I_\nu (\mathbf{n}) + \eta_\nu / 4\pi - \sigma_\nu I_\nu (\mathbf{n}) + \sigma_\nu \int \phi(\mathbf{n}; \mathbf{n}') I_\nu (\mathbf{n}') d\Omega' \]

Transfer is solved along a long ray across the domain

- Physical quantities are interpolated at each grid
- A bit complex coding
- No numerical diffusion (accuracy equivalent to long char.)
- Operations (same as short char.)

\[ \sim N_x N_y N_z \cdot N_\theta N_\phi N_\nu \]
Calculations of Radiation Tensor

Specific intensities \( \mathcal{I} \) (in the phase space)

Integration in the momentum space

Radiation tensor \( R^{\mu\nu} \) in the nearest neighbors of \( \mathcal{I} \) (in the position space)

\[
R^{\mu\nu} = \int k^\mu k^\nu \mathcal{I} \, dP
\]

Eddington tensor \( f_{ij} \)

Grid in position space

Affine parameter

Geodesics
Tetrad formalism

Mix-frame approach

LNRF (locally non-rotating reference frame )
(Local Minkovski spacetime)  

\[
(T^\mu_\nu + R^\mu_\nu)_{,\nu} = F^\mu \quad \iff \quad (T^{\alpha\beta} + R^{\alpha\beta})_{;\beta} = F^\alpha
\]

Global curved spacetime

\[
R^{\alpha\beta} = \varepsilon^\alpha_\mu \varepsilon^\beta_\nu R^{\mu\nu}
\]

Conservation Law of RHD

\[
T^{\mu\nu} = (\rho_0 + \rho_0 \varepsilon/c^2 + P/c^2)u^\mu u^\nu - P\eta^{\mu\nu}
\]

\[
R^{\mu\nu} = \begin{pmatrix} E & F/c \\ F/c & P \end{pmatrix}
\]

\[
E = \frac{1}{c} \int_0^\infty d\nu \int I_\nu d\Omega : \text{radiation energy density}
\]

\[
F = \int_0^\infty d\nu \int I_\nu n d\Omega : \text{radiation flux}
\]

\[
P = \frac{1}{c} \int_0^\infty d\nu \int I_\nu nn d\Omega : \text{radiation stress tensor}
\]
Coordinates in Curved Space

Horizon capture

Boyer-Lindquist coordinate

Kerr-Schild coordinate
Static test: BH shadow

A source is located behind a BH. A shadow forms by light bending + frame-dragging + gravitational redshift.

Takahashi & Umemura 2014, in prep
Time Evolution of Invariant Brightness

Intensity

Angle

Time
Capture of Wave Front

Geodesics Patterns and Grids in Phase Space

A. angular coordinates at initial points 
\((\theta_i, \phi_i)\)

B. angular coordinates at end points 
\((\theta_e, \phi_e)\)

C. angular coordinates in LNRF at initial points 
\((\bar{\theta}, \bar{\phi})\)
(direction of geodesics)

D. angular coordinates measured from \((\theta_i, \phi_i)\) 
\((\theta_*, \phi_*)\)

# Geodesics are calculated for \(0 \leq \theta_i \leq \pi, 0 \leq \phi_i \leq 2\pi\) \(-3\pi \leq \phi_e \leq 3\pi\) for \(0 \leq \bar{\phi} \leq 2\pi\)

1. all possible patterns of geodesics in phase space.
2. relativistic orbits with \(-3\pi \leq \phi_e \leq 3\pi\) are included.
Ray-tracing calculation

Light ray rotates 5 times around a BH
Dynamical Test 1: Wave-front propagation

GR-Boltzmann calculation

$E_{\text{rad}} \text{ (a.u.)}$

$t=0.0$
Wave fronts can cross each other without any collision!

Takahashi & Umemura 2014, in prep
Dynamical Test 2: Photon wave front from a rotating hot spot

Boltzmann calculations

\[ t = 0.0 \]

\[ E_{rad} \text{ (a.u.)} \]

\[
\begin{array}{ccc}
0.1 & 1 & 10 & 100 \\
0.01 & 0.1 & 1 \\
\end{array}
\]

\[ x [\text{GM/c}^2] \]

\[ y [\text{GM/c}^2] \]
Boltzmann calculations

Ray-tracing calculations

\[ N_i (r) = 90, \quad N_j (\phi) = 256 \]

Geodesics = 4608
Vermeer
Master of light

Variable Eddington-tensor Radiation-hydrodynamics with Metric Enchained Ray-tracing

General Relativistic Radiation Transfer
Radiation Hydrodynamics in Curved Space
HA-PACS (PACS-VIII) system

- System spec.
  - 268 + 64 (TCA) nodes
  - CPU 89+28.7TFLOPS + GPU 713T+335.4FLOPS =802+364TFLOPS = total 1.166PFLOPS
  - Memory 34TByte, memory bandwidth 26TByte/sec
  - Bi-section bandwidth 2.1TByte/s
  - Storage 504TByte
  - Power 408kW

True GPU-direct
With cooperation of NVIDIA
GR radiative transfer solver “Vermeer”

The time-dependent Boltzmann equation of photons is directly integrated along the geodesics. The propagation of wave fronts is solved in proper time, so that the causality is completely retained.

Tests in a Kerr metric
(1) BH shadow
(2) Wave front propagation
(3) Propagation of radiation wave from a rotating hot spot

Moment calculations
Eddington tensor of radiation in LNRF (locally non-rotating reference frame) is transformed to that in the curved space, which can close GR-RHD equations properly.

Parallel calculations
The scheme can be parallelized with GPUs.