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Dynamical captures of two non-spinning equal mass black holes in weakly hyperbolic orbits

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Outline

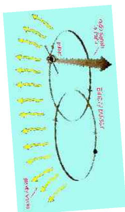
I. Introduction
II. Gravitational radiation capture
III. Post-Newtonian results
IV. NR results
V. Discussion

I. Introduction

• How many bound systems will be formed through encounters?

"Unbound" ➔ "Bound"

• This "Capturing cross-section" is important to understand evolutions of stars & black holes after their formations, event rate of GW detection, etc..



- **Binary black hole simulations so far:**
  - Mostly for quasi-circular orbits (i.e., e ~ 0)

- **Detectability including eccentric mergers:**
  - How many binary BHs to be formed?:
    - O'Leary, Kocsis, Loeb ('09) for a galaxy w/ a SMBH at the center
    - Based on the PN approximation with parabolic orbit approximations for weakly hyperbolic orbits

$$\Gamma_{1GN} = \int_{r_{min}}^{r_{max}} dr\, 4\pi r^2 \int_{M_{min}}^{M_{max}} dM \int_{M_{min}}^{M} dm \times \iint_{x_m, x_M > 10, J > J_{LC}} d^3v_m\, d^3v_M\, f_m(r, v_m) f_M(r, v_M) \sigma_{cs} w,$$

$$\sim 10^{-8} \text{ and } 10^{-10}\ \text{yr}^{-1}$$

➔ 1~1000/yr at aLIGO!

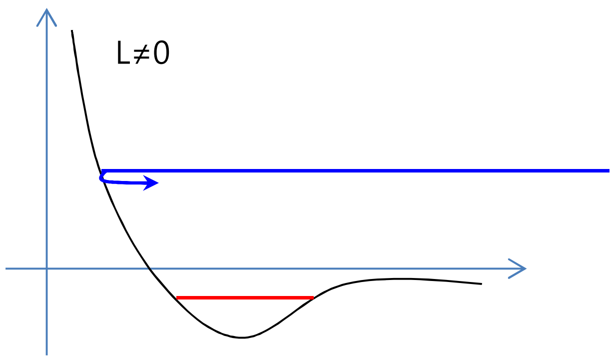
N-body simulations (Hong & Lee ('13)): ~ 0.02~20/yr

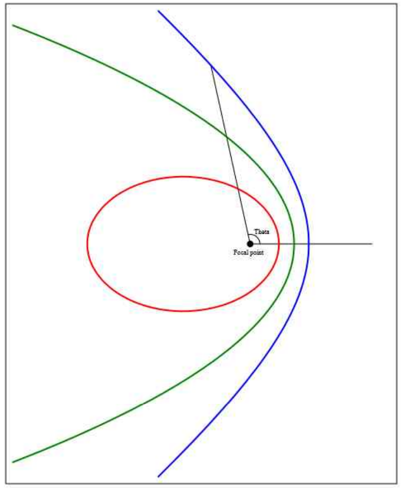
  - East et al ('13):
    - Developed a new strategy taking into account the characteristic features of eccentric merger sources.

- **It might be interesting to study the capturing processes in full GR, and see where the approximations break down.**

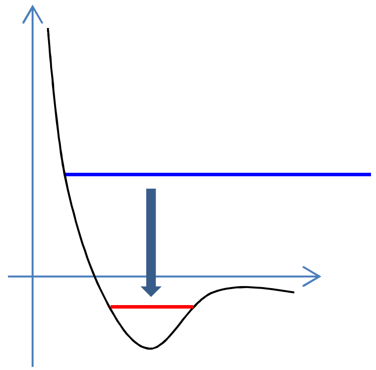
II. Gravitational radiation capture

- **Newtonian gravity for two-body interactions:**

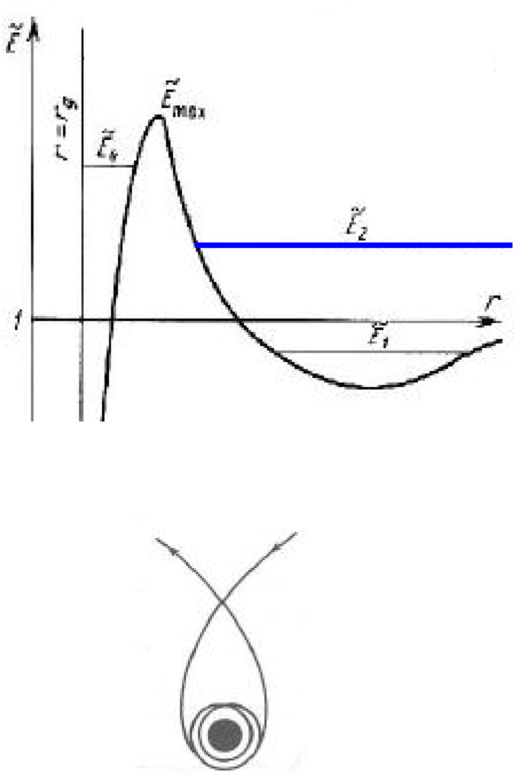




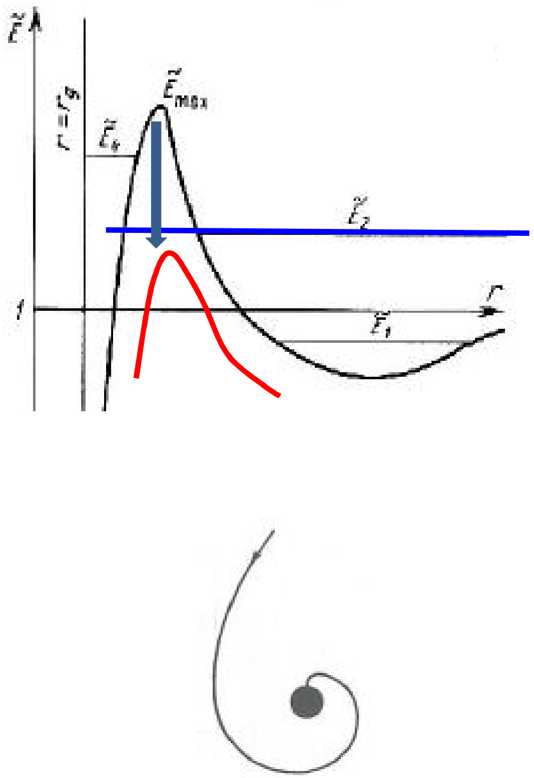
- As long as L≠0, an "unbound" object initially should escape to infinity after an encounter.
- For captures, e.g., "Hyperbolic orbit ➔ Elliptic orbit", a third object is needed to extract out a suitable energy of the system;



- **A geodesic motion on the Schwarzschild BH background:**

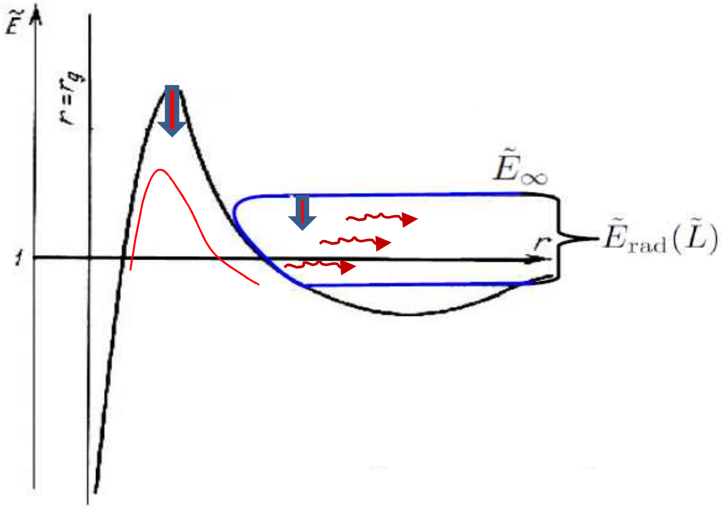


Escape to infinity
after encountering



Direct capturing could occur
even if L≠0

- **General relativity:** "Gravitational wave (GW) emissions"

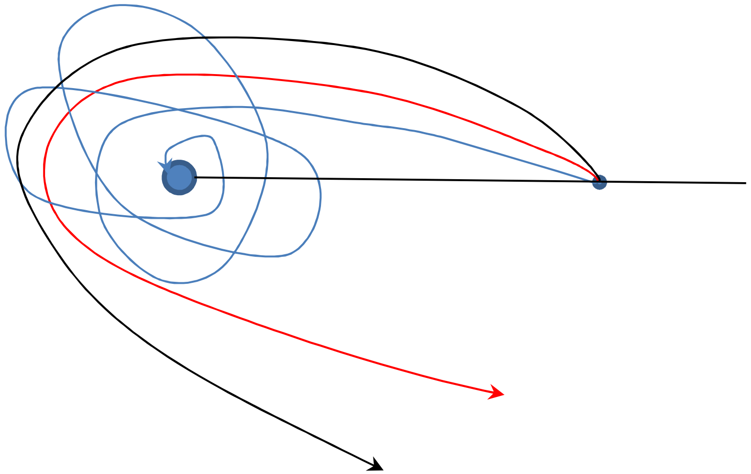


- So, initial "unbound" to bound process could happen through an encounter, forming a bound binary!!

➔ Gravitational radiation capture

or

Dynamical capture



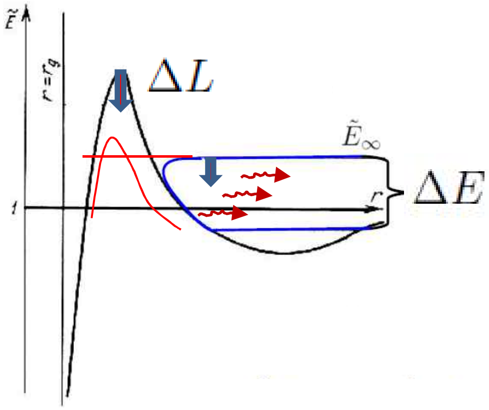
- The marginal capturing gives

$$b_{\rm max} = \frac{L_{\rm cr}(E)}{\mu v_\infty} = \frac{L_{\rm cr}(E)}{\sqrt{2\mu E}}$$

$$\sigma_{\rm cap} = \pi b_{\rm max}^2$$

III. Post-Newtonian results

- **Emitted energy (2.5PN) for a hyperbolic orbit:** R. Hansen ('72)



$$\Delta E = -\frac{2}{15}\frac{G^{7/2}}{c^5} {m_1}^2 {m_2}^2 \left(m_1 + m_2\right)^{1/2} \times \frac{(\pi - \theta_0)(96 + 292e^2 + 37e^4) + \frac{1}{3} e \sin\theta_0 (602 + 673e^2)}{a^{7/2}(e^2-1)^{7/2}}$$

$$\Delta L = -\frac{8}{5}\frac{G^3}{c^5} {m_1}^2 {m_2}^2 \frac{(\pi - \theta_0)(8 + 7e^2) + e \sin\theta_0 (13 + 2e^2)}{a^2(e^2-1)^2}$$

$$a = Gm_1m_2/(2E) \qquad e = \sqrt{1 + 2EL^2/(G^2\mu {m_1}^2 {m_2}^2)} \qquad \cos\theta_0 = \frac{1}{e}$$

$$\Delta E(E,\, e) = \Delta E(E,\, L) = \Delta E(E,\, b)$$

$$\Delta L(E,\, e) = \Delta L(E,\, L) = \Delta L(E,\, b)$$

$$L = b \times \mu v_\infty = b\sqrt{2\mu E}$$

$|\Delta E| > E$ ⇨ CAPTURED

$|\Delta E| = E$ ⇨ JUST CAPTURED

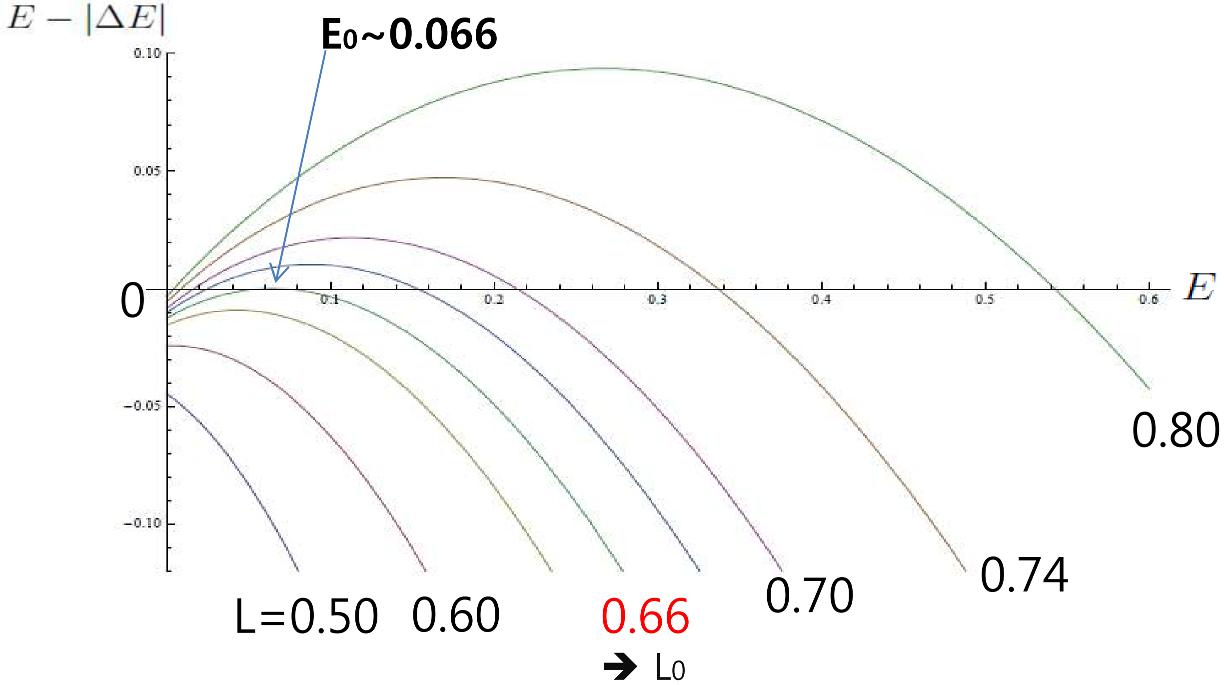
$|\Delta E| < E$ ⇨ ESCAPE TO INFINITY

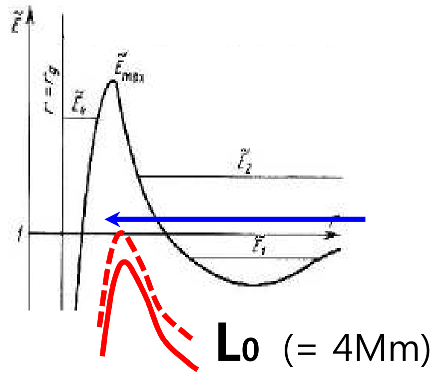
$\Delta E(E,\, b_{\rm max}) = -E$ ➡ $b_{\rm max} = b_{\rm max}(E)$ $= \dfrac{L_{\rm cr}(E)}{\sqrt{2\mu E}}$

For L = fixed with $m_1=m_2=1/2$,

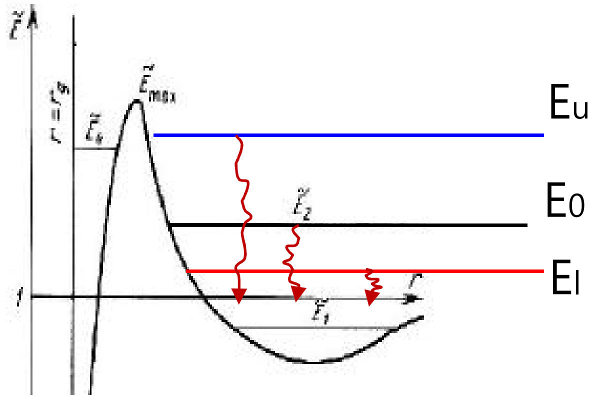
$$\Delta E \sim -\frac{85\pi}{393216L^7} - \frac{61\pi}{2560L^5}E - \frac{37\pi}{120L^3}E^2 + \cdots \quad \text{for} \quad E \sim 0$$

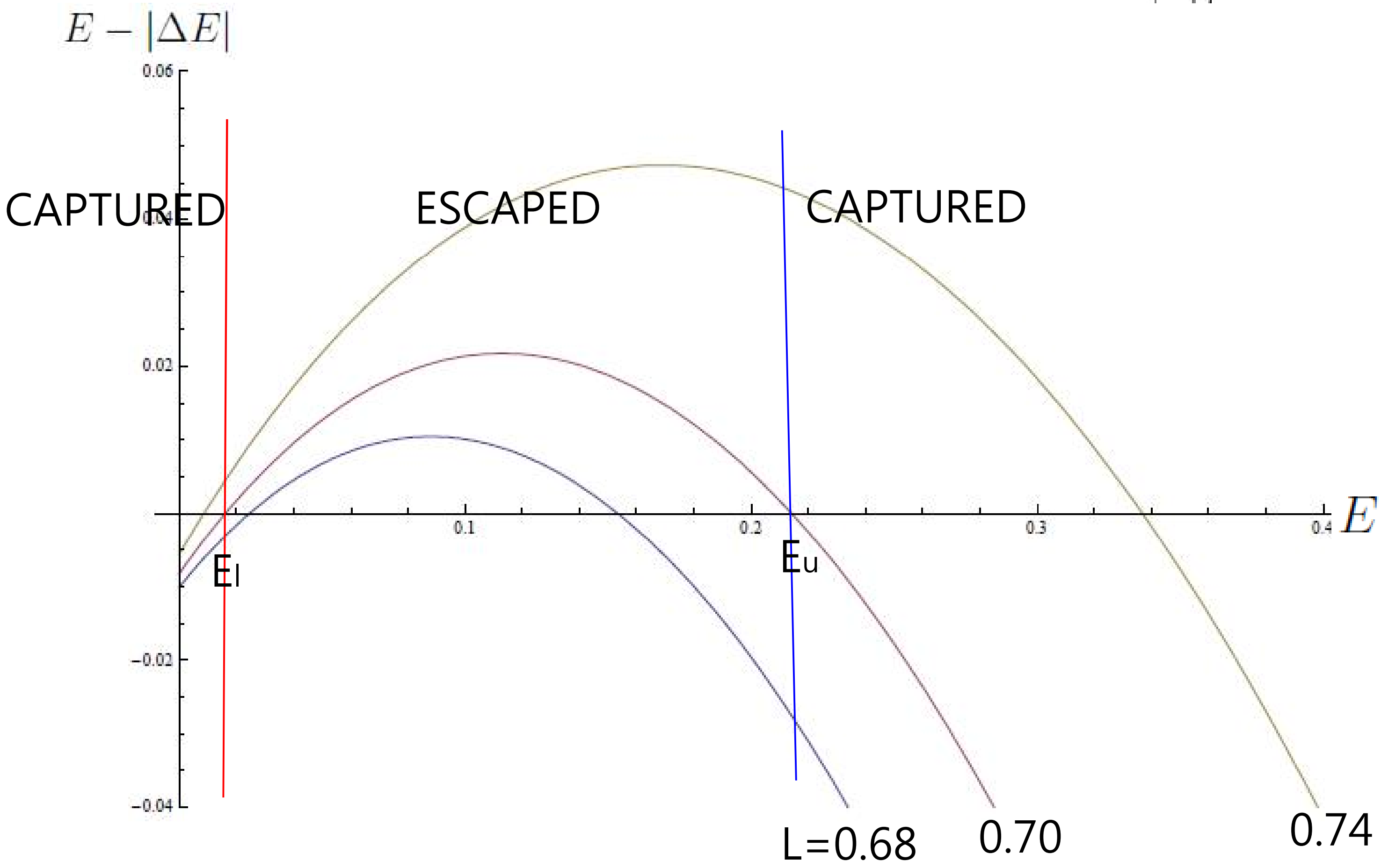
$$\sim -\frac{37\pi}{240L^3}E^2 - \cdots \quad \text{for} \quad E \sim \infty$$



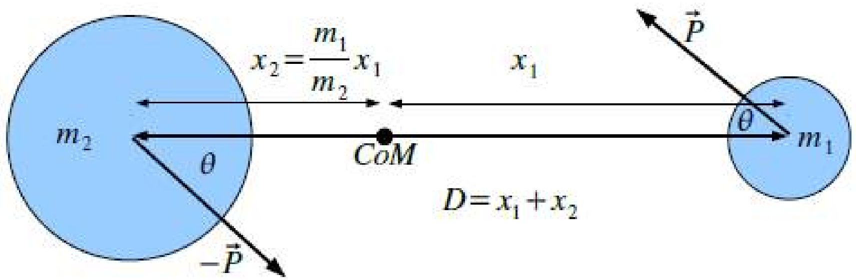


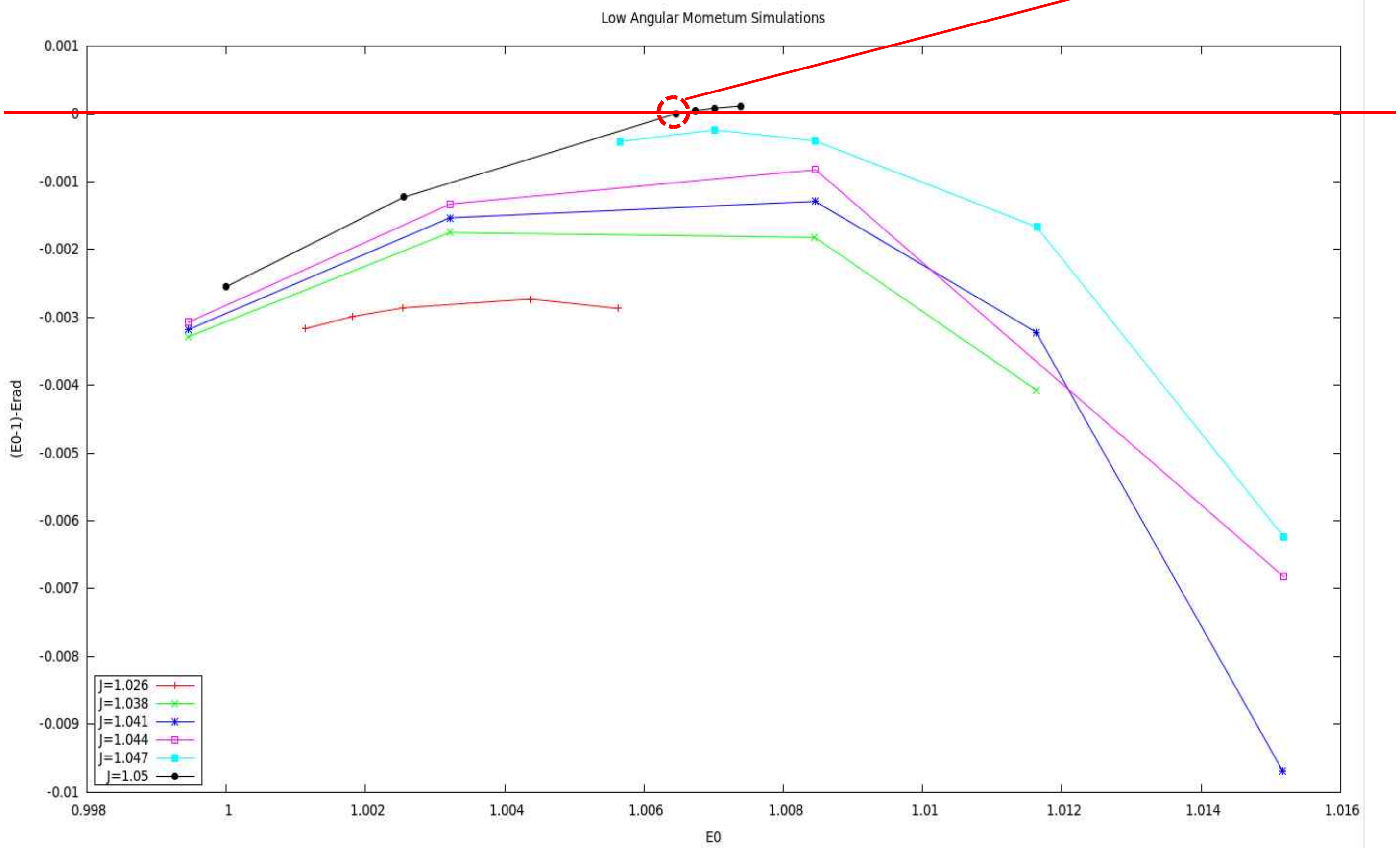
All initial unbound objects will be captured directly if $L \leq L_0$.


Eu
E0
El


$E - |\Delta E|$
0.06
0.02
-0.02
-0.04
0.1
0.2
0.3
0.4
E
CAPTURED
ESCAPED
CAPTURED
El
Eu
L=0.68
0.70
0.74

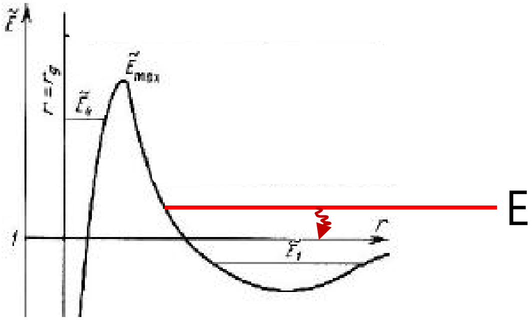
- **NR simulations** for two non-spinning equal mass black holes:





Identifying this point is computationally very expensive!

• **Parabolic approximation:**



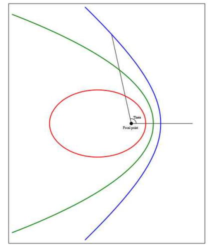
- Instead of finding E for which

$$\Delta E(E, L) = -E,$$

we use that

$$\Delta E(E,\ L) \sim \Delta E(E = 0,\ L)$$

for a weakly hyperbolic orbit, i.e., "small" E ~ 0 (e~1).



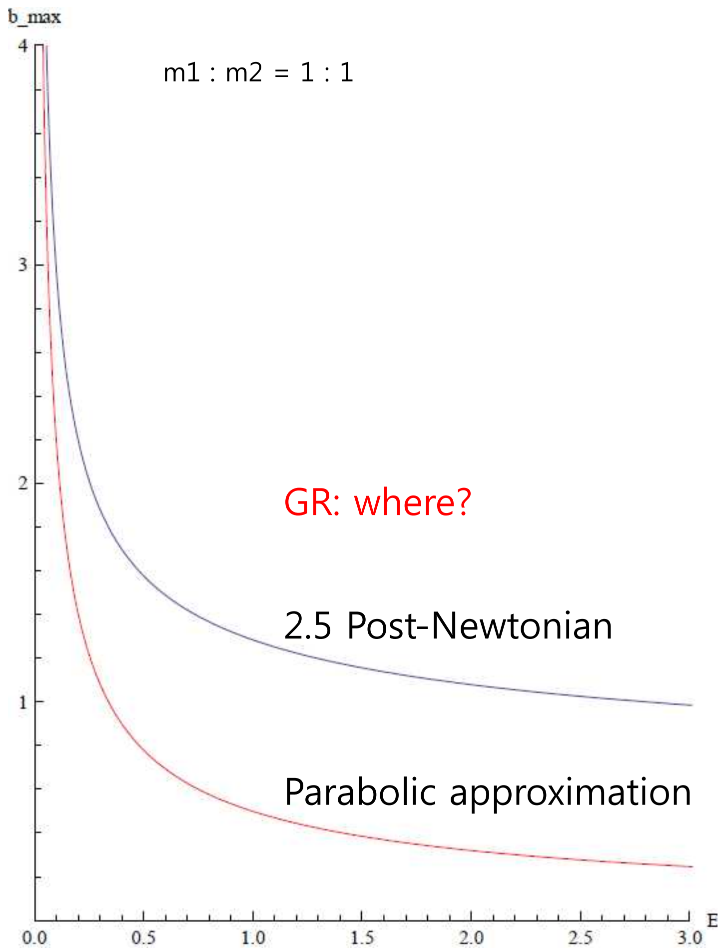
- Namely, we prepare an initial data for two black holes in a parabolic orbit, and perform the numerical simulation to get the emitted energy ΔE_{para}.

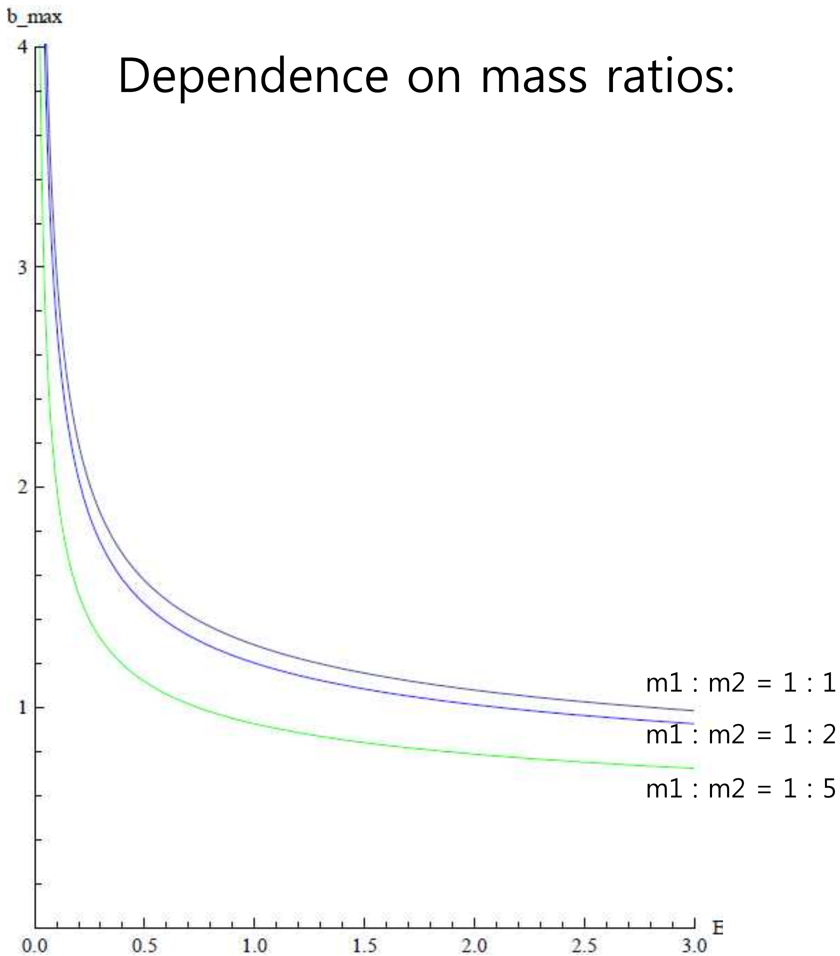
- Then, we set E = $|\Delta E_{para}|$.

- For such parabolic initial data, we use the EOB formulation.

- For most simulations, the parabolic approximation was used to save computational time.
- We also checked the validity of this approximation as E increases.

• **Maximum impact parameter or capturing cross-section:**

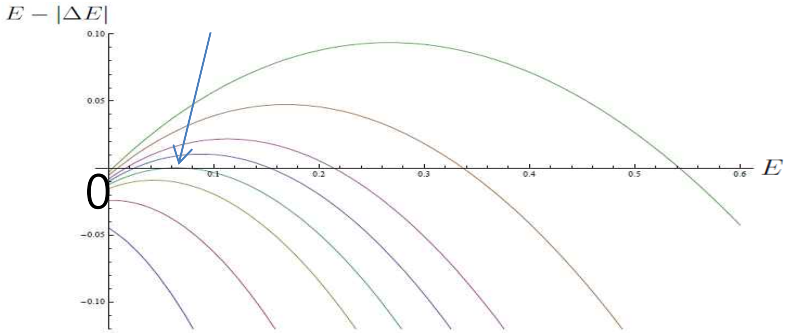


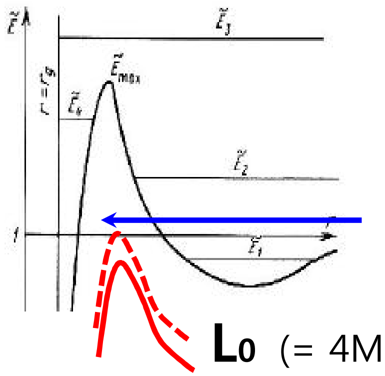


w/ m1 + m2 = 1

Ⅳ. NR results

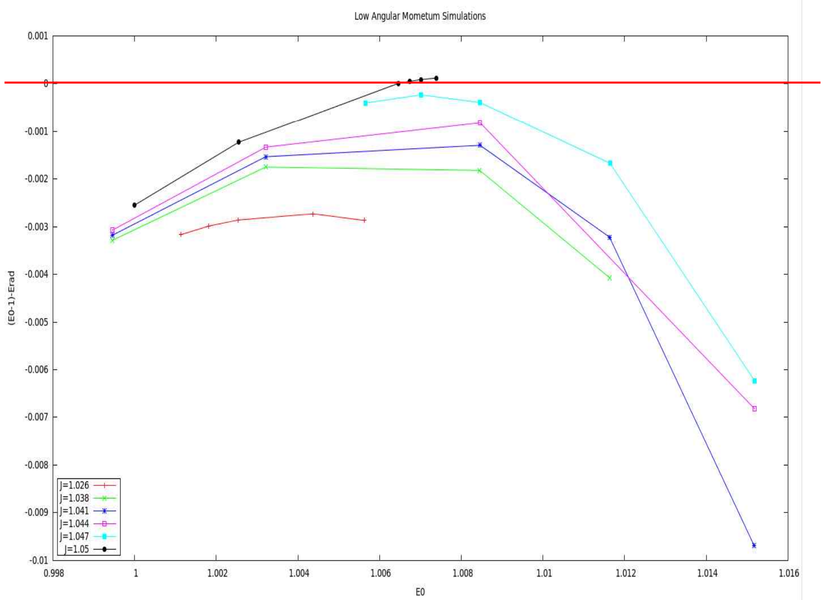
- **Minimum angular momentum giving escapes to infinity:**





L0 (= 4Mm)

L0~0.66



(M=m=1/2)

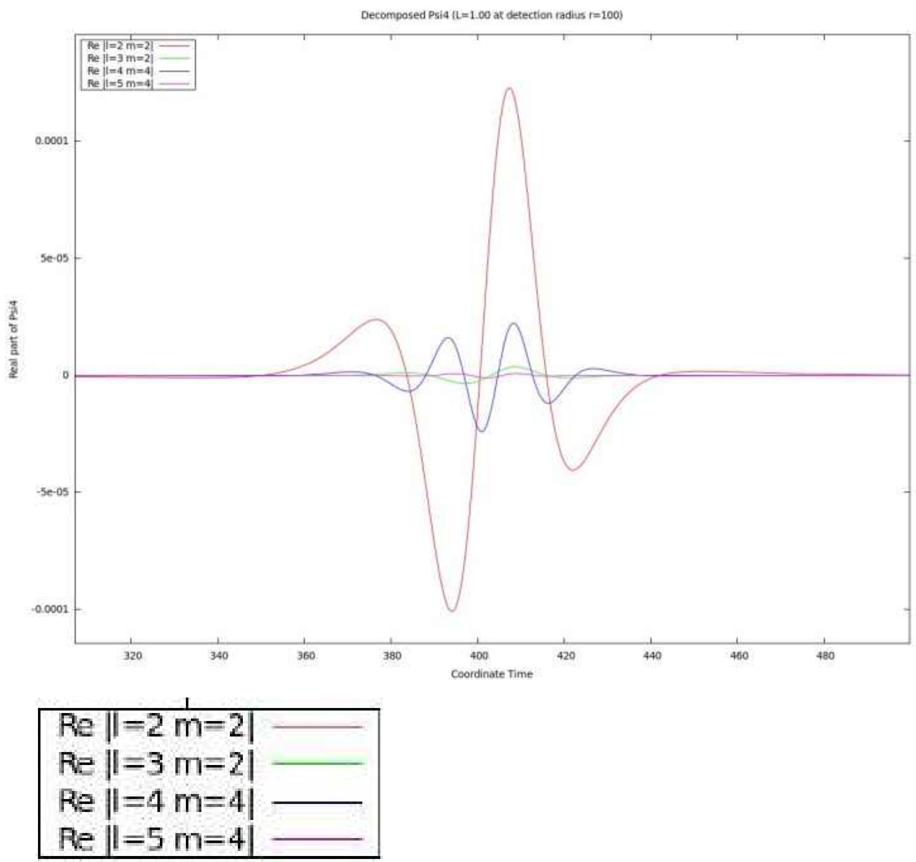
L0 =
- 0.0 in Newtonian Gravity
- [ ] in 2.5PN
- 1.0 in geodesic motions
- [ ] in GR

- **Features of orbits and waveforms:** See the talk by Y. Bae!

- **Multi mode contributions:**

$$\Psi_4 = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} A^{l,m} \left({}_{-2}Y^{l,m}(\theta, \phi) \right)$$

Decomposed Psi4 (L=1.00 at detection radius r=100)



$$\frac{dE}{dt} = \lim_{r\to\infty} \frac{r^2}{16\,\pi} \oint \left| \int_{-\infty}^{t} \Psi_4 \, dt' \right|^2 d\Omega$$

$$= \lim_{r\to\infty} \frac{r^2}{16\,\pi} \sum_{l,m} \left| \int_{-\infty}^{t} A^{l,m} \, dt' \right|^2$$

Energy budget (L=1.00):

l=2:
l=3:
l=4:
Rest.

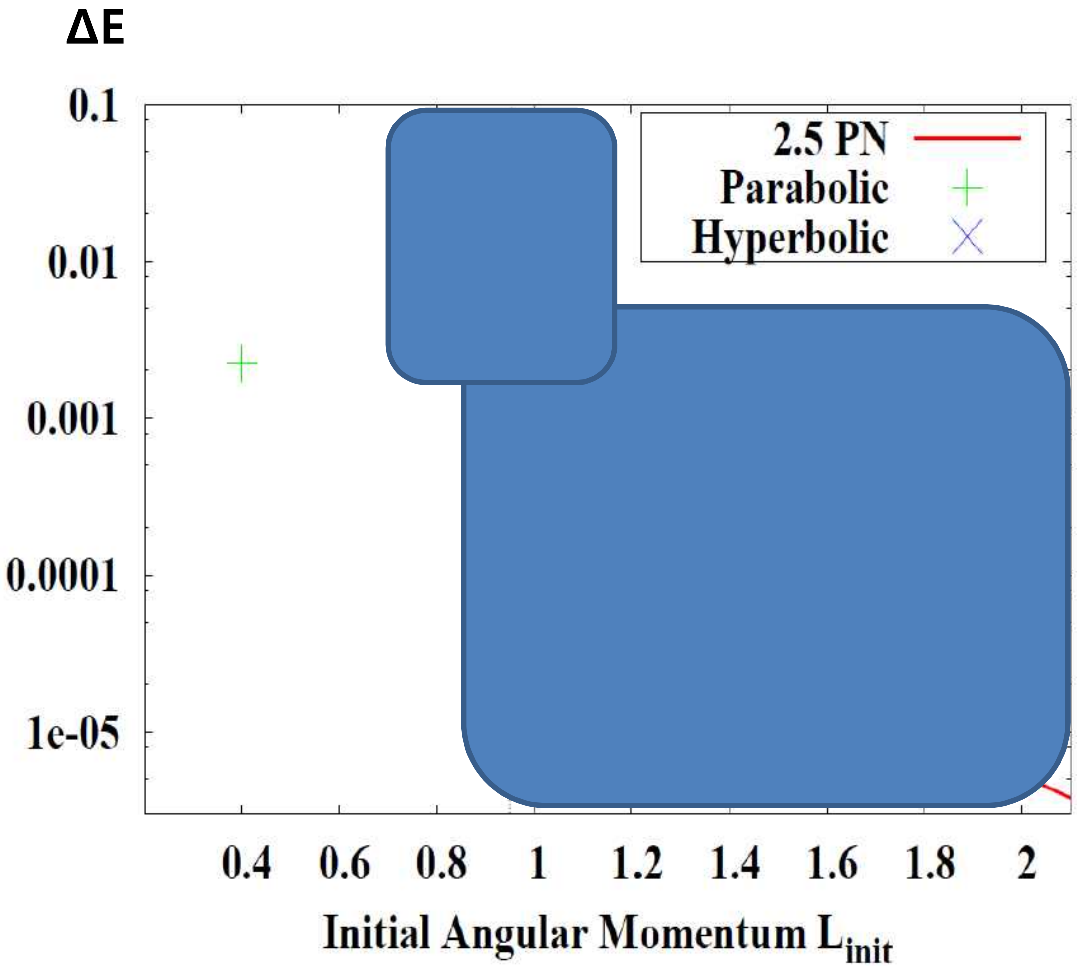
- **Energy emitted by GWs for marginal captures: ΔE**

- Large radiation for small initial angular momentum

- at the peak, but less than mostly (~3.7% for e~0)

- 2.5PN is in good agreement w/ NR for large L, but it breaks down as L decreases.

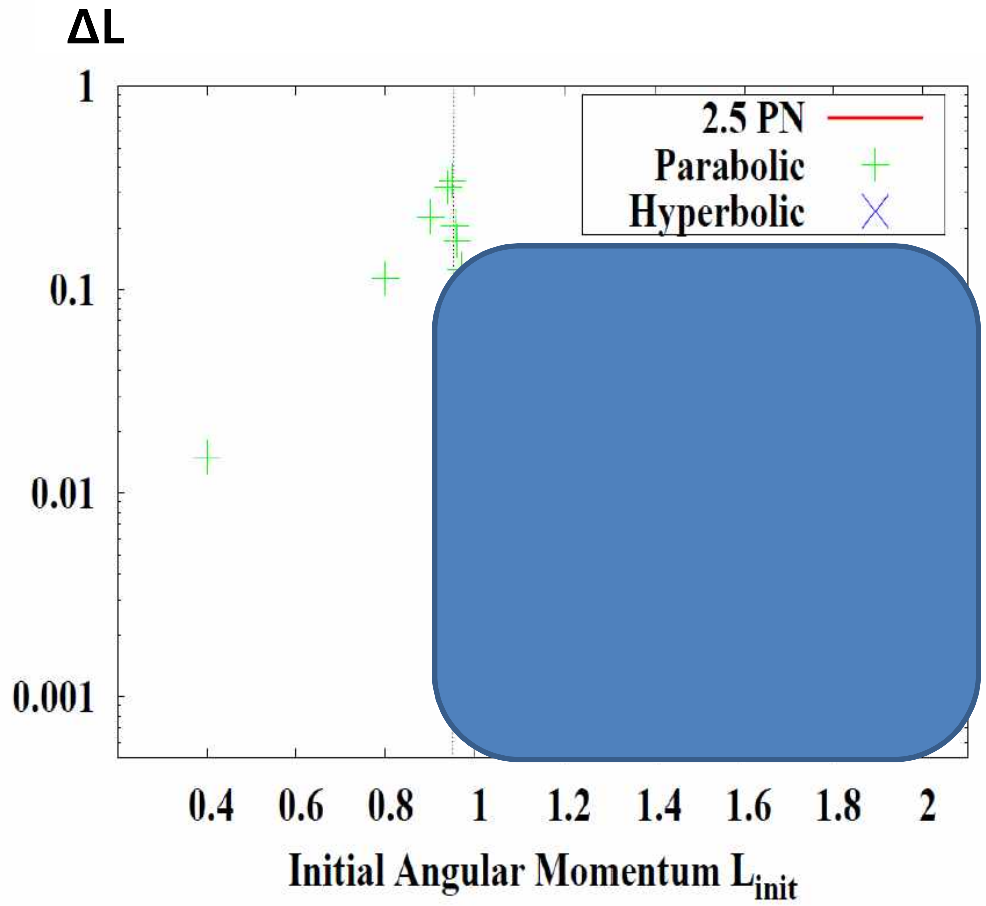
- The parabolic approximation breaks down for .



- **Angular momentum radiation: ΔL**

- Similar features with the energy radiation

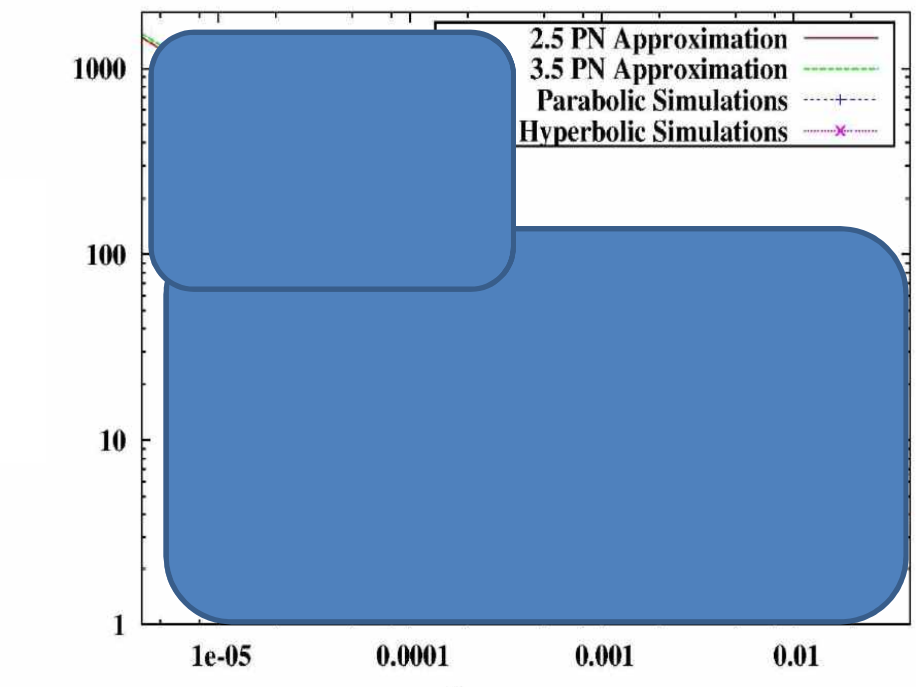
- at the peak



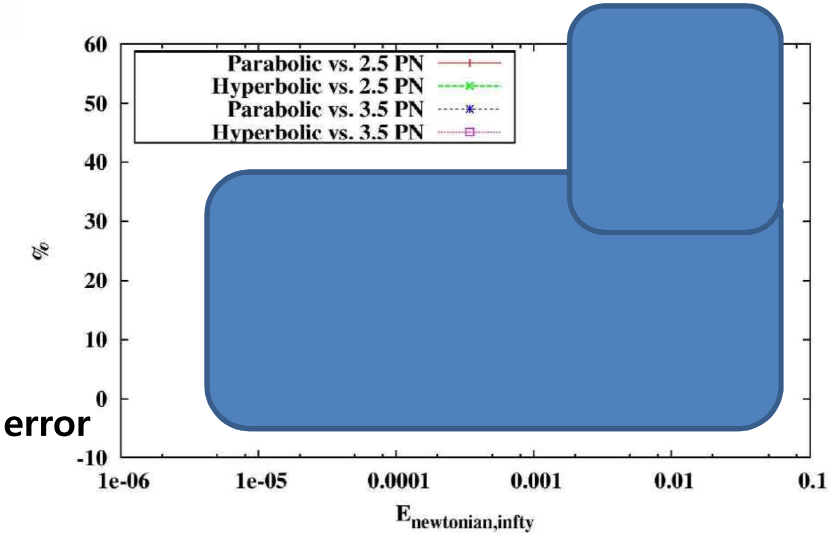
- **Maximum impact parameter or capturing cross-section:**

  - Less capturing for large initial energies
  - 2.5PN deviates from NR as E increases:
  -
  - For any given energy, the GR result gives the strongest capturing.

b_{max}



Percentage error



V. Discussion

- Gravitational radiation capture processes of two equal mass BHs are investigated in full GR for the first time, and the capture cross-section is compared with the PN calculations.
- There are many interesting regions in the parameter space unexplored, including un-equal masses and spins. For un-equal masses, non-quadrupole (i.e., $l > 2$) mode contributions might NOT be negligible.
- How much is the event rate improved in GW detection experiments due to the GR effect?
- In eccentric merger simulations, it is interesting to understand the mechanism of radiations and its efficiency in more detail.