A Review of Primary and Recent GRMHD Simulations

Shinji Koide (Kumamoto University)

I will show primary and recent results of general relativistic magnetohydrodynamics (GRMHD) simulations.

• Ideal GRMHD: Primary results ← our group
  Recent results ← other groups at US and UK.
• Resistive GRMHD: magnetic reconnection near a black hole ← our group
Outline

• Motivation: Relativistic jets in universe
• Generalized GRMHD and ideal GRMHD
• Review of ideal GRMHD simulations
  – Primary and recent calculations:
    • Jet formation from accretion disk around non-rotating black hole
    • Magnetically energy extraction of rotating black hole
    • Relativistic, magnetically-driven jet formation
• Resistive GRMHD simulations
  – Calculation of magnetic reconnection near black hole
• Summary and future prospects
Motivation: Relativistic Jets in Universe

AGN Quasar (QSO)
- Radio lobe
- Relativistic jet
- UV and optical radiation
- Accretion disk (~10⁵ km)
- Spinning supermassive black hole
- Mirabel, Rodriguez 1998

Several ~ 10⁴ ~ 10⁶ light years

Micro-quasar (QSO)
- Radio lobe
- Relativistic jet
- Companion star
- Acceleration of plasma

Collimation of outflow

Force of acceleration and collimation:
1) Magnetic field force
2) Radiation pressure
3) Gas pressure

Several ~ 3 light years

Gamma-ray burst (GRB)
- C-rays
- Relativistic jet
- ~ 1 AU
- ~ 1 km

Force of acceleration and collimation:
→ Magnetically-driven jet

~ light years
Magnetically driven jet from a disk around a star: A numerical simulation of non-relativistic ideal MHD
Kudoh, Matsumoto & Shibata (1998)

To confine such fast rotating plasma, the central object should be a black hole.

Non-relativistic numerical simulations and theories show:

$$v_j \approx v_{\text{disk}}$$

Relativistic jet:

$$v_j \approx c \quad v_{\text{disk}} \approx c$$
Black Hole Magnetosphere

- Corona
- Magnetic Field Lines
- Plasma
- Ergosphere
- Strong current: inertia & momentum
- Frame-dragging effect
- General relativistic plasma
  - "Generalized GRMHD"
  - Resistive GRMHD
  - Ideal GRMHD

- Black Hole Magnetosphere
- Plasma Disk
- Rotating Black Hole
General relativistic two-fluid equations

We assume two fluids where each fluid is constructed by particles with mass \( m_\pm \) and charge \( \pm e \).

\[
\nabla_v \left( n_\pm U_\pm^\nu \right) = 0
\]

Conservation of particle number

\[
\nabla_v \left( h_\pm U_\pm^\mu U_\pm^\nu \right) = -\nabla^\mu p_\pm \pm en_\pm U_\pm^\nu F_\mu^\nu \pm R^\mu
\]

Conservation of momentum and energy

Friction force density of two fluids

\[
h_\pm = h(\rho_\pm, p_\pm) = \rho_\pm \hat{h}(p_\pm / \rho_\pm)
\]

Equation of state

\[
\nabla_v \ast F^{\mu\nu} = 0
\]

Dual tensor

\[
\nabla_v F^{\mu\nu} = J^\mu
\]

Maxwell equations

Unit system: \( c = 1, \ \varepsilon_0 = 1, \ \mu_0 = 1 \)
Definition of one-fluid-type variables

\[ \rho = m_+ n_+ \gamma_+ + m_- n_- \gamma_- \]  
\[ n = \rho / m \]  
\[ U^\mu = \frac{1}{\rho} \left( m_+ n_+ U_+^\mu + m_- n_- U_-^\mu \right) \]  
\[ J^\mu = e \left( n_+ U_+^\mu - n_- U_-^\mu \right) \]  
\[ p = p_+ + p_- \quad \Delta p = p_+ - p_- \]  
\[ h = n^2 \left( \frac{h_+}{n_+^2} + \frac{h_-}{n_-^2} \right) \]  
\[ \mu = m_+ m_- / m^2 \]  
\[ \Delta \mu = (m_+ - m_-) / m \]  
\[ h^\# = h - \Delta \mu \Delta h \quad \Delta h^\# = \Delta \mu h - \frac{1 - 3 \mu}{2 \mu} \Delta h \]
Generalized GRMHD equations
≡ Two-fluid equations ≡ One-fluid-type equations

Koide (2010)

\[ \nabla_v \left( \rho U^v \right) = 0 \]
4-velocity

Equation of continuity

\[ \nabla_v \left[ p g^{\mu \nu} + h U^\mu U^\nu \right] + \frac{\mu h}{(ne)^2} J^\mu J^\nu + \frac{\Delta h}{2ne} \left( U^\mu J^\nu + J^\mu U^\nu \right) + T_{EM}^{\mu \nu} = 0 \]
Conservation of momentum and energy

Energy-momentum tensor of fluid \( T_{hyd}^{\mu \nu} \)

\[ \frac{1}{ne} \nabla_v \left[ \frac{\mu h}{ne} \left( U^\mu J^\nu + J^\mu U^\nu \right) + \frac{\Delta h}{2} U^\mu U^\nu - \frac{\mu \Delta h}{(ne)^2} J^\mu J^\nu \right] \]

Thermo-electromotive force

\[ \frac{1}{2ne} \nabla^\mu (\Delta \mu - \Delta p) + \left( \frac{\Delta \mu}{ne} J^\nu \right) F_{\nu}^\mu - \eta \left[ J^\mu + \left( U^\nu J^\nu \right) (1 + \Theta) U^\mu \right] \]

Field strength tensor

\[ \nabla_v F^{\mu \nu} = J^\mu \]

Hall effect

\[ \nabla_v \ast F^{\mu \nu} = 0 \]
Maxwell equations

\[ T_{EM}^{\mu \nu} = -\frac{1}{4} g^{\mu \nu} F_{\rho \sigma} F^{\rho \sigma} + F^\mu \sigma F^{\nu \sigma} \]
Energy-momentum tensor of field

Resistivity

Inertia and momentum of charge and current

Standard Ohm's law

Generalized general relativistic Ohm's law
Simplest case: Covariant form of ideal GRMHD equations

\[ \nabla_v \left( nU^\nu \right) = 0 \]  
(equation of continuity)

\[ \nabla_v T^\mu^\nu = 0 \]  
(energy and momentum)

\[ F_{\mu\nu} U^\nu = 0 \]  
(ideal MHD condition)

\[ \nabla_v \star F^{\mu\nu} = 0 \quad \nabla_v F^{\mu\nu} = -J^\mu \]  
(Maxwell equations)

Metric of space-time around a rotating black hole:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \sum_{i=1}^{3} \left( h_i dx^i - \alpha \beta^i dt \right)^2 = -dt^2 + \sum_{i=1}^{3} (dx^i)'^2 \]

Lapse function

Shift vector \[ \beta = (\beta^1, \beta^2, \beta^3) \]

gravitational time delay, gravitational potential

velocity of dragged frame
3+1 Formalism of Ideal GRMHD Equations
~ similar to nonrelativistic ideal MHD

\[
\frac{\partial D}{\partial t} = \nabla \cdot \left[ \alpha D \left( \hat{v} + \beta \right) \right] \\
\quad \text{(conservative form)} \\
\quad \text{(conservation of particle number)}
\]

\[
\frac{\partial \hat{P}}{\partial t} = \nabla \cdot \left[ \alpha \left( \hat{T} + \beta \hat{P} \right) \right] - (D + \varepsilon) \nabla \alpha + \alpha f_{\text{curv}} - \hat{P} : \\
\quad \text{(equation of motion)} \\
\quad \text{(special relativistic effect)}
\]

\[
= h\gamma^2 \hat{v} + \hat{E} \times \hat{B} \\
\hat{T} = h\gamma^2 \hat{v} + \left( p + \frac{\hat{B}^2}{2} + \frac{\hat{E}^2}{2} \right) I - \hat{B} \hat{B} - \hat{E} \hat{E}
\]

\[
\frac{\partial \varepsilon}{\partial t} = \nabla \cdot \left[ \alpha \left( \hat{P} - D \hat{v} + \varepsilon \beta \right) \right] - \hat{P} \cdot \nabla \alpha - \hat{T} : \sigma \\
\quad \text{(equation of energy)}
\]

\[
\frac{\partial \hat{B}}{\partial t} = -\nabla \times \left[ \alpha \left( \hat{E} - \beta \times \hat{B} \right) \right] \\
\quad \text{(flux conservation)}
\]

\[
\nabla \cdot \hat{B} = 0 \\
\rho_e = \alpha \nabla \cdot E \\
\hat{E} + \hat{v} \times \hat{B} = 0 \\
\quad \text{(ideal MHD condition)}
\]

\[
\nabla \cdot \hat{E} = 0 \\
\nabla \cdot \hat{B} = 0 \\
\sigma : \text{shear of } \beta
\]

Similarity with MHD has boosted the recent development of ideal GRMHD simulations!

No coupling with other Eqs.
First, primary GRMHD simulation of jet formation around non-rotating black hole:

GRMHD version of Newtonian MHD simulation of jet formation like Kudoh et al. (1998)

Koide, Shibata, & Kudoh (1999)
First, primary GRMHD simulation with rotating black hole

(Koide, Shibata, Kudoh, Meier 2002)

(1) Kerr black hole: maximally rotating rotation parameter, $a = J/J_{\text{max}} = 0.9999$

(2) Magnetic field: Uniform around Kerr black hole (Wald solution)

(3) Plasma: zero momentum, uniform, low density and pressure $\rho_0 = 0.1 B_0^2/c^2$, $p_0 = 0.06 \rho_0 c^2$

(4) Axisymmetry, symmetry with respect to equatorial plain
Kerr black hole, Uniform magnetic field, No Accretion disk

\[ \frac{R}{r_S} = 0 \]

\[ r_S = \frac{2GM_\bullet}{c^2} \]

Lines: Magnetic field surfaces
Arrows: Velocity of plasma

Koide, Shibata, Kudoh, Meier 2002
\[ t = 1\tau_s \]
\[ \tau_s = \frac{r_s}{c} \]

Lines: Magnetic field surfaces
Arrows: Velocity of plasma

- \( B \)
Lines: Magnetic field surfaces
Arrows: Velocity of plasma

Kerr black hole

$R/r_S$

$z/r_S$

$t = 7 \tau_S$

$-B$

$c$
Power Radiation along Magnetic Field Lines cross Ergosphere, but No Outflow

Kerr black hole

Ergosphere

Magnetic field lines

Propagation of Alfven wave: Electromagnetic energy transportation

$t = 7 \tau_s$

Formation of Negative Energy and outward Alfven wave → Extraction of black hole rotational energy

\[ t = 7 \tau_S \]

**Energy-at-infinity density**

\[ \epsilon^\infty = \hat{\alpha} \hat{e} + \omega_\phi L \]

Total energy at Fiducial observer frame

**Arrow:** Power flux density

**Solid line:** Magnetic field surface

“MHD Penrose process”

(Hirotani, et al. 1992)

Black hole energy extraction with negative energy of plasma, which induced by magnetic force.
First long term simulation of uniformly magnetized plasmas around Kerr BH

(Komissarov 2005)

“MHD Blandford-Znajek mechanism”: Purely electromagnetic mechanism extracts rotation energy of the black hole.
Causal extraction of black hole energy due to electromagnetic field

\[ \hat{E} = -\hat{v}_{F\perp} \times \hat{B} \quad \left( \hat{v}_{F\perp} \perp \hat{B} \right) \]

Energy transport through the horizon:

\[ e_{EM}^{\infty} = \alpha \hat{B}^2 \left[ \frac{1}{2} \left( 1 + (v_{F\perp})^2 \right) + \beta \cdot \hat{v}_{F\perp} \right] \]

\[ S_{EM}^P = \alpha e_{EM}^{\infty} (\hat{v}_{F\perp} + \beta) \]

Angular-velocity of field lines at horizon

Angular-velocity of frame at horizon

\[ S_{EM}^P = R_H^2 \Omega_F (\Omega_F - \Omega_H^2) (\hat{B}_{PH})^2 (\hat{v}_{F\perp} + \beta) \]

Ideal (GR)MHD

Force-free

Radiation in vacuum

When \( 0 < \Omega_F < \Omega_H \), energy is transported outward through the horizon.

GRMHD: “MHD Blandford-Znajek mechanism”

force-free: Original (FF) Blandford-Znajek mechanism

Electromagnetic wave

\[ S_{EM}^P = \Omega_H \frac{m}{\omega - m\Omega_H} (\alpha \hat{B})^2 (\hat{v}_{F\perp} + \beta) \]

When \( \omega - m\Omega_\phi < 0 \), energy is transported outward through the horizon.

Electromagnetic wave: “Superluminal radiation”
More realistic magnetic configuration with accretion disk around black hole

Black Hole Magnetosphere

Black Hole

Ergosphere

Plasma Disk

Corona

Magnetic Field Lines

Plasma

Black Hole Magnetosphere
First longer term simulation of ideal GRMHD with accretion disk around rotating black hole

Formation of Relativistic Jet
Ideal GRMHD longer-term simulation

Lorentz factor: $\Gamma > 5$ ($v_{\text{jet}} = 0.98c$)

$t = 7000r_s / c$

$\alpha = 0.9375$

Color: log $\rho$

Lines: magnetic surfaces

- J. C. McKinney 2006
Longitudinal structure of relativistic jet

Jet collimation

Jet acceleration

$\Gamma \approx 7$

$v_{\text{jet}} \approx 0.99c$

• J. C. McKinney 2006
Recent result of GRMHD simulations:
3D long term GRMHD simulation

McKinney & Blandford (2009)
Almost the same initial condition of McKinney (2006)

Jet is stable against kink (current-driven) instability, in spite of strong twist of magnetic filed lines.

Relativistic jet propagates stably against Kelvin-Helmholtz instability and kink (current-driven) instability.
Schematic picture of induction of strong magnetic field by frame-dragging effect

- **Rapidly Rotating Black Hole**
- **Ergosphere**
- **Frame-dragging effect**
- **Increase in magnetic pressure**
- **Current loop**
- **Increase in magnetic pressure**
- **Disk rotation**
- **Magnetic bridge**
Twist of magnetic bridge by ergosphere

Blow off plasma → Outflow

Magnetic bridge

Rapidly Rotating Black Hole

Increase in magnetic pressure

Disk rotation

Plasma

~ Magnetic tower model proposed by D. Lynden-Bell
Schematic picture of phenomena caused by the magnetic bridge near the black hole

Present result (Ideal GRMHD)

Magnetic field

Sub-relativistic jet

Anti-parallel magnetic field is formed

Note: magnetic bridge between ergosphere and disk can not keep stationary, and explosively expands. Magnetic bridge between ergosphere and disk is “open bridge”!
Magnetic reconnection must be caused near black hole horizon spontaneously.

GRMHD with nonzero resistivity (Resistive GRMHD)
Covariant form of standard resistive GRMHD equations

- General relativistic equations of conservation laws:
  \[ \nabla^\mu (h \dot{U}_\mu) = 0 \] (particle number)
  \[ \nabla^\mu T^{\mu \nu} = 0 \] (energy and momentum)

Maxwell equations:

\[ \nabla^\nu \star F_{\mu \nu} = 0 \]

Ohm’s law with resistivity:

\[ F_{\mu \nu} U^\nu = \eta \left( J_\mu + (U_\nu J^\nu)U_\mu \right) \]
3+1 Formalism of Resistive GRMHD Equations

(conservative form)

(conervation of particle number)

Special relativistic mass density,

\[
\frac{\partial D}{\partial t} = -\nabla \cdot \left[ \alpha D (\hat{v} + \beta) \right]
\]

general relativistic effect

Special relativistic total momentum density

\[
\frac{\partial \hat{P}}{\partial t} = -\nabla \cdot \left[ \alpha \left( \hat{T} + \beta \hat{P} \right) \right] - (D + \varepsilon) \nabla \alpha + \alpha f_{\text{curv}} - \hat{P} : \sigma
\]

special relativistic effect

\[
\hat{P} = h \gamma \hat{v} + \hat{E} \times \hat{B}
\]

Maxwell equations

(conservative form)

Special relativistic total energy density

\[
\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot \left[ \alpha \left( \hat{P} - D \hat{v} + \varepsilon \beta \right) \right] - \hat{P} \cdot \nabla \alpha - \hat{T} : \sigma
\]

\[
\frac{\partial \hat{B}}{\partial t} = -\nabla \times \left[ \alpha \left( \hat{E} - \beta \times \hat{B} \right) \right] - \alpha (\hat{J} + \rho_e \beta) + \frac{\partial \hat{E}}{\partial t} = \nabla \times \left[ \alpha \left( \hat{B} + \beta \times \hat{E} \right) \right]
\]

(Maxwell equations)

\[
\nabla \cdot \hat{B} = 0, \quad \rho_e = \alpha \nabla \cdot \hat{E}
\]

\[
\hat{E} + \hat{v} \times \hat{B} = \frac{\eta}{\gamma} \left[ \hat{J} - \gamma^2 \left( \rho_e - (\hat{v} \cdot \hat{J}) \right) \hat{v} \right]
\]

(Ohm’s law with finite resistivity)

We treat Ampere’s law as an equation of time evolution of electric field (Watanabe & Yokoyama 2006).
Magnetic reconnection in split-monopole magnetic field around black holes

- Schwarzschild black hole:

Petschek like fast magnetic reconnection is found even with uniform resistivity!

S. Koide & R. Morino (2013)
A Toy model of the fast magnetic reconnection in ergosphere: Mechanism of “Daruma-Otoshi”

Daruma-otoshi: Japanese traditional toy. When one piece is hit to shit out, the pieces upper and lower are connected.
Summary

• I reviewed primary and recent results of ideal GRMHD simulations.

• Ideal GRMHD simulations provide insights on the extraction mechanism of black hole rotational energy (“MHD Penrose process”, “MHD Blandford-Znajek mechanism”).

• Recent long term ideal GRMHD simulations revealed that strong magnetic field is induced spontaneously around the rapidly rotating black hole. The strong magnetic field across ergospheres launches relativistic jets.

• During jet formation, anti-parallel magnetic field configuration are formed spontaneously. In the region, the magnetic reconnection will take place near the black hole.

• We showed a simple test case of the magnetic reconnection near the black hole using resistive GRMHD code.
Future prospects:

• To investigate the magnetic reconnection around a rotating black hole, we should use the resistive GRMHD equations.

• Furthermore, the generalized GRMHD equations including terms of inertia effect of current, thermal electromotive force and Hall effect may be required. When the current density is extremely large, the inertial effect of electron may be dominant.

• Our generalized GRMHD equations identical to two-fluid approximation may be useful to analyze the modification.