Relativistic Electromagnetic Two-fluid Simulations of Pulsar Wind Termination Shocks

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Outline

• Relativistic two-fluid model.
• Application of 1D simulations to the sigma problem of pulsar winds. [Amano & Kirk, 2013]
• Extension to multidimensions.
Relativistic MHD eqs.

Mass Conservation Law
\[ \partial_{\mu} (\rho u^{\mu}) = 0 \]

Energy-Momentum Conservation Law
\[ \partial_{\mu} \left( T_{m}^{\mu\nu} + T_{f}^{\mu\nu} \right) = 0 \]

Maxwell eqs.
\[ \partial_{\mu} F^{\mu\nu} = -J^{\nu} \]
\[ \partial_{\mu} F^{*\mu\nu} = 0 \]

Matter
\[ T_{m}^{\mu\nu} = \rho h u^{\mu} u^{\nu} + p \eta^{\mu\nu} \]

EM Field
\[ T_{f}^{\mu\nu} = F_{\lambda}^{\mu} - \frac{1}{4} F^{\lambda \kappa} F_{\lambda \kappa} \eta^{\mu\nu} \]

Ohm’s law
\[ F^{\mu\nu} u_{\nu} = 0 \]
Relativistic two-fluid eqs.

Mass Conservation Law
\[ \partial_\mu \left( \rho_s u^\mu_s \right) = 0 \]

Energy-Momentum Conservation Law
\[ \partial_\mu T^\mu_\nu_s = - \frac{q_s}{m_s} \rho_s u^\mu_s F^\mu_\nu \]
\[ \partial_\mu F^{\mu_\nu} = - \sum \frac{q_s}{m_s} \rho_s u^\nu_s \]
\[ \partial_\mu F^{*\mu_\nu} = 0 \]

Maxwell eqs.
\[ T^\mu_\nu_s = \rho_s h_s u^\mu_s u^\nu_s + p_s \eta^{\mu_\nu}_s \]
\[ T^\mu_\nu_f = F^\mu_\lambda - \frac{1}{4} F^{\lambda_\kappa} F_{\lambda_\kappa} \eta^{\mu_\nu} \]

Ohm's law is no longer needed as the EM-field evolves according to Maxwell eqs with the electric current calculated from contributions of each species.
What are the differences?

\[
\frac{1}{c} \frac{\partial}{\partial t} \mathbf{B} = -\nabla \times \mathbf{E} \quad \frac{1}{c} \frac{\partial}{\partial t} \mathbf{E} = \nabla \times \mathbf{B} - \frac{4\pi}{c} \mathbf{J}
\]

In MHD, Ampere’s law is redundant because of the frozen-in assumption:

\[
\mathbf{E} = -\frac{\mathbf{V}}{c} \times \mathbf{B}
\]

The current density does not appear explicitly in the equation (when written in the conservative form.)

In two-fluid (or Vlasov-Maxwell system), the current density is defined by the sum of contributions from each components.

\[
\mathbf{J} = \sum \frac{q_s}{m_s} \rho_s \mathbf{u}_s
\]
MHD waves appear around the origin.
Important differences from MHD:
• Dispersion around the inertial length of each species.
• High frequency waves (both electrostatic and electromagnetic) above the plasma frequency.
The sigma problem of Crab nebula

- The spin-down luminosity of a pulsar is carried away in a form of a relativistic wind. It is launched as a high-sigma wind, while observations imply the opposite.
- Similar Poynting-flux dominated flows may be found in other high-energy astrophysical environments as well.

\[ \sigma = \frac{B^2}{4\pi \gamma^2 n m c^2} \]

\[ \sigma \ll 1 \text{ @ nebula} \]
\[ \sigma \gg 1 \text{ @ wind} \]
The striped pulsar wind

- Series of current sheets (i.e., MHD waves) are produced by obliquely rotating pulsars.
- Magnetic reconnection has been believed to be important for the dissipation in the wind zone.
Pulsar rotation frequency

- The rotation frequency of a young pulsar (measured in the lab. frame) can be higher than the local proper plasma frequency in a far wind zone.

\[ \frac{\omega_p}{\Omega} \sim 2.8 \times 10^6 \left( \frac{\dot{N}}{10^{40} \text{s}^{-1}} \right) \left( \frac{L}{10^{38} \text{ergs/s}} \right)^{-1/2} \left( 1 + \sigma \right)^{1/2} \left( \frac{r}{r_L} \right)^{-1} \]

Fiducial parameters for the Crab.

Interaction between the shock and the upstream waves is likely to occur in non-MHD regime, then what happens?

Termination Shock

MHD regime (overdense)

non-MHD regime (underdense)
Parametric instability

\[ \omega_\pm = \omega_0 \pm \omega \quad k_\pm = k_0 \pm k \]

- The strong pump EM wave can couple to a longitudinal perturbation (sound-like wave) when the matching condition is satisfied. The generated longitudinal waves will eventually dissipate through various processes (formation of shocks, collisionless damping).
Simulation setup

- A highly magnetized pair plasma is continuously injected from the left boundary with a supersonic flow speed.
- The plasma carries a circularly polarized magnetic shear wave (i.e., entropy mode).
- Simulations are performed in the shock rest frame (by properly setting up density/temperature in the downstream).

* 1D simulations with HLL scheme and WENO5 reconstruction.
High freq. v.s. Low freq.

- Parameters
  - $\sigma = 10$, $\gamma = 40$, $\Omega/\omega_p = 1.2, 0.4$
- An extended precursor ahead of a subshock is formed associated with the dissipation.
- The structure remarkably resembles that of a cosmic-ray modified shock [Drury & Völk'81].
- The modification is due to strong superluminal waves.
The incoming entropy-mode wave has already been converted into superluminal waves in the precursor, which subsequently decay into sound-like waves.
Downstream sigma

- The magnetization parameter sigma substantially decreases through the precursor and subshock.
- The remaining Poynting flux in the downstream is entirely carried by superluminal waves, meaning that the frozen-in condition does not mean anything.
Extension to Multidimensions

How to satisfy the divergence constraints?

\[ \nabla \cdot \mathbf{E} = 4\pi \rho_c \quad \nabla \cdot \mathbf{B} = 0 \]

Can it be used as an alternative to RMHD?
Summary of Scheme

• HLL flux for fluid variables.
• HLL-UCT (2D HLL flux at edge center) for EM field.
• Piecewise linear reconstruction with MC2 for interpolation.
• 3rd order TVD Runge-Kutta method.

• Caveat: No “standard” test problems
  – Only a few multidimensional codes (for pair plasmas) have been reported in the literature. [Zenitani+2009, Barkov+2014]
HLL-UCT scheme

Upstream Constrained Transport [Londrillo & Del Zanna 2004, Del Zanna 2007] may also be used for the full Maxwell equations. Normal component of E, and B are located at face center for each direction.

1. Interpolate E, B to cell center.

\[
\hat{E}_z = \frac{E^NE_z + E^SE_z + E^NW_z + E^SW_z}{4} - \frac{c}{2} (B^S_x - B^N_x) + \frac{c}{2} (B^W_y - B^E_y)
\]

\[
\hat{B}_z = \frac{B^NE_z + B^SE_z + B^NW_z + B^SW_z}{4} + \frac{c}{2} (E^S_x - E^N_x) - \frac{c}{2} (E^W_y - E^E_y)
\]

face center where normal E, B are constant.

4. Ez, Bz at edge center are calculated as 2D HLL fluxes.

5. Normal component of particle number flux at each face is used for defining current density.
Brio-Wu Problem

Two-fluid model with a small inertial length (< grid size) is able to reproduce the ideal MHD result. (N = 800)
Orszag-Tang Vortex

Relativistic analog of Orszag-Tang vortex problem often used for benchmark test of multidimensional MHD codes. The resolution $192 \times 192$ is the same as Beckwith & Stone (2011). Again, the result (with inertia length < grid size) is consistent with MHD.

Density

Density [Beckwith & Stone 2011]
Density for an electron-ion plasma (\(m_i/me = 25\), and \(\lambda_i = 0.01\))

Both div(E) and div(B) errors are kept within the machine epsilon.
An overdense (by a factor of 100) and overpressure (by a factor of 2000) region is setup initially inside a circle $r<0.8$, which expands into the surrounding medium. [c.f., Komissarov 1999, Del Zanna+2007]

Moderately magnetized case: $B_x = 0.1$ ($\beta = 0.1$)
Cylindrical Explosion

Strongly magnetized case: \( B_x = 1.0 \) (\( \beta = 0.001 \))

Many RMHD codes are unable to run this strongly magnetized (i.e., low beta) case unless some ad-hoc techniques are used.
Magnetic Reconnection

Relativistic analog of GEM magnetic reconnection problem (Birn+2001) in a relativistic current sheet. Resistivity is implemented as a friction term [Zenitani+2009]

\[
m_i/m_e = 25, \quad \sigma_i = 1.0, \quad \sigma_e = 25
\]

\[
T_i \sim m_i c^2, \quad T_e \sim m_e c^2
\]

256 \times 512 grids
Summary

• Relativistic electromagnetic two-fluid model is literally a tool to study the dynamics of relativistic plasmas of intermediate scale between PIC and MHD.

• However, it can potentially be used also for macroscopic phenomena as a better alternative to MHD.

• Extension to higher order schemes (WENO, MP5, etc.) is currently ongoing.