Magnetorotational processes in core-collapsed supernovae

Sergey Moiseenko, Gennady Bisnovatyi-Kogan
Space Research Institute,
Moscow, Russia
Core-collapsed supernovae are brightest events in the Universe.

- **Crab nebula**
- **Cas A**
- **SN1987A**
Magnetorotational mechanism for the supernova explosion was suggested by Bisnovatyi-Kogan (1970)

(original article was submitted: September 3, 1969)

Amplification of magnetic fields due to differential rotation, angular momentum transfer by magnetic field. Part of the rotational energy is transformed to the energy of explosion
Basic equations: MHD + self-gravitation, infinite conductivity:

\[
\begin{align*}
\frac{dx}{dt} &= \mathbf{u}, \quad \frac{d\rho}{dt} + \rho \text{div}\mathbf{u} = 0, \\
\rho \frac{du}{dt} &= -\text{grad}\left(p + \frac{\mathbf{H} \cdot \mathbf{H}}{8\pi}\right) + \frac{1}{4\pi} \text{div}(\mathbf{H} \otimes \mathbf{H}) - \rho \text{grad}\Phi \\
\rho \frac{d\varepsilon}{dt} + p\text{div}\mathbf{u} + \rho F(\rho, T) &= 0, \quad p = P(\rho, T), \quad \varepsilon = E(\rho, T), \\
\Delta \Phi &= 4\pi G \rho, \\
\rho \frac{d}{dt}\left(\frac{\mathbf{H}}{\rho}\right) &= \mathbf{H} \cdot \nabla \mathbf{u}.
\end{align*}
\]

Additional condition \text{div}\mathbf{H}=0

Axis symmetry \(\frac{\partial}{\partial \phi} = 0\) and equatorial symmetry \((z=0)\) are supposed.

Notations:

\[
\begin{align*}
\frac{d}{dt} &= \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla, \quad \mathbf{x} = (r, \phi, z), \quad \mathbf{u} - \text{velocity}, \quad \rho - \text{density}, \quad p - \text{pressure}, \\
\mathbf{H} - \text{magnetic field}, \quad \Phi - \text{gravitational potential}, \quad \varepsilon - \text{internal energy}, \\
G - \text{gravitational constant}.
\end{align*}
\]
Magnetorotational supernova in 1D

Bisnovaty-Kogan et al. 1976, Ardeljan et al. 1979

\[ t_{\text{explosion}} = \frac{1}{\sqrt{\alpha}}, \quad \left( \alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}} \right) \]

Example: \( \alpha = 10^{-2} \Rightarrow t_{\text{explosion}} = 10 \),

\( \alpha = 10^{-12} \Rightarrow t_{\text{explosion}} = 10^6 \)

FIG. 3. Shape of a field line in the region near the core at the time \( t_\alpha = 7 \) for \( \alpha = 10^{-2} \) (dashed line) and \( \alpha = 10^{-4} \) (solid line).
(Thanks to E.A. Skiba)
Presupernova Core Collapse

Equations of state take into account degeneracy of electrons and neutrons, relativity for the electrons, nuclear transitions and nuclear interactions. Temperature effects were taken into account approximately by the addition of radiation pressure and an ideal gas.

Neutrino losses were taken into account in the energy equations.

A cool white dwarf was considered at the stability limit with a mass equal to the Chandrasekhar limit.

To obtain the collapse we increase the density at each point by 20% and we also impart uniform rotation on it.
Operator-difference scheme (finite volumes)

Ardeljan & Kosmachevskii Comp. and Math. Modeling 1995, 6, 209 and references therein

Lagrangian, implicit, triangular grid with rezoning, completely conservative

Method of **basic** operators (Samarskii) – grid analogs of basic differential operators:

- \( \text{GRAD} \text{(scalar)} \text{ (differential)} \sim \text{GRAD} \text{(scalar)} \text{ (grid analog)} \)
- \( \text{DIV} \text{(vector)} \text{ (differential)} \sim \text{DIV} \text{(vector)} \text{ (grid analog)} \)
- \( \text{CURL} \text{(vector)} \text{ (differential)} \sim \text{CURL} \text{(vector)} \text{ (grid analog)} \)
- \( \text{GRAD} \text{(vector)} \text{ (differential)} \sim \text{GRAD} \text{(vector)} \text{ (grid analog)} \)
- \( \text{DIV} \text{(tensor)} \text{ (differential)} \sim \text{DIV} \text{(tensor)} \text{ (grid analog)} \)

Implicit scheme. Time step restrictions are **weaker** for implicit schemes (no CFL condition).

The scheme is Lagrangian=> conservation of **angular** momentum.
Operator-difference scheme for hydro equations

\[ \rho_i V_i = \rho_i^0 V_i^0 = m_i, \quad \forall \Delta_i \in \omega_\Delta \]

\[ x_{jt} = u_j^{(0.5)}, \quad \forall \overline{x_j} \in \omega_x \]

\[ \rho_j u_{jt} = - (\nabla \times g)_j - q \rho_j \left( S_x (\nabla \Phi) \right)_j, \quad \forall \overline{x_j} \in \omega_x \]

\[ \rho_i \varepsilon_{it} = - g_i \left( \nabla \Delta \cdot u^{(0.5)} \right)_i, \quad \forall \Delta_i \in \omega_\Delta \]

\[ g = p^{(\alpha)} + \omega, \quad \omega = - \frac{\nu}{\eta}, \quad \forall \Delta_i \in \omega_\Delta \]

\[ \eta_i = 1 / \rho_i = T_i / p_i, \quad \varepsilon_i = T_i / (\gamma - 1), \quad \forall \Delta_i \in \omega_\Delta \]

\[ (\nabla \times \cdot \nabla \Phi)_j = \rho_j, \]

\[ \rho_j = (S_x \rho)_j = \frac{1}{3 V_x} \sum_{k=1}^{K_j} \rho_k V_k^\Delta, \quad \forall \overline{x_j} \in \omega_x. \]

where \[ u_{jt} = \left\{ u_{jt}^r - \frac{u_j^{r(0.5)}}{r_j^{(0.5)}} u_j^{r(0.5)}, \quad u_{jt}^\varphi + \frac{u_j^{\varphi(0.5)}}{r_j^{(0.5)}} u_j^{r(0.5)}, \quad u_{jt}^z \right\} \]
Example of calculation with the triangular grid
Initial magnetic field –quadrupole-and dipole-like symmetries
Temperature and velocity field

Specific angular momentum

Magnetorotational explosion for the quadrupole-like magnetic field
Magnetorotational explosion for the \textit{dipole-like} magnetic field
Time evolution of different types of energies

![Graph showing the time evolution of different types of energies: E\textsubscript{kinpol}, E\textsubscript{rot}, E\textsubscript{magpol}, and E\textsubscript{magtor}.](image_url)
Ejected energy and mass

Ejected energy $0.6 \cdot 10^{51} \text{ erg}$

Ejected mass $0.14M_\odot$

Particle is considered “ejected” – if its kinetic energy is greater than its potential energy.
Magnetorotational explosion for the different $\alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}} = 10^{-2} - 10^{-12}$


\[ \frac{E_{\text{mag}0}}{E_{\text{grav}0}} = 10^{-2} - 10^{-12} \]
Dependence of the explosion time from $\alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}}$

$t_{\text{explosion}} \sim -\log(\alpha)$  \hspace{1cm} (for small $\alpha$)

$\alpha = 10^{-6} \Rightarrow t_{\text{explosion}} \sim 6,$

Example:

$\alpha = 10^{-12} \Rightarrow t_{\text{explosion}} \sim 12.$
MRI in 2D – Tayler instability

Tayler, R. MNRAS 1973
Toy model for MDRI in the magnetorotational supernova

\[ \frac{dH_\phi}{dt} = H_r \left( r \frac{d\Omega}{dr} \right); \] at the initial stage of the process \( H_\phi < H^*_\phi \): \( H_r \left( r \frac{d\Omega}{dr} \right) \approx \text{const}, \)

beginning of the MRI => formation of multiple *poloidal* differentially rotating vortexes

\[ \frac{dH_r}{dt} = H_{r0} \left( \frac{d\omega_v}{dl} \right), \] in general we may approximate: \( \left( \frac{d\omega_v}{dl} \right) \approx \alpha (H_\phi - H^*_\phi) \).

Assuming for the simplicity that \( r \frac{d\Omega}{dr} = A \) is a constant during the first stages of MRI, and taking \( r \) as a constant we come to the following equation:

\[ \frac{d^2}{dt^2} \left( H_\phi - H^*_\phi \right) = AH_{r0} \alpha (H_\phi - H^*_\phi) \]

\[
\begin{aligned}
H_\phi &= H^*_\phi + H_{r0} e^{\sqrt{A\alpha H_{r0}}(t-t^*)}, \\
H_r &= H_{r0} + \frac{H_{r0}^{3/2} \alpha^{1/2}}{\sqrt{A}} \left( e^{\sqrt{A\alpha H_{r0}}(t-t^*)} - 1 \right).
\end{aligned}
\]
MDRI
\[ B_0 = 10^{(9)} \text{G} \]

No MDRI
\[ B_0 = 10^{(12)} \text{G} \]
MDRI development

Toroidal to poloidal magnetic energy relation
3D simulations

Mikami H., Sato Y., Matsumoto T., Hanawa, T.  

recently
P. Mosta et al. MR supernovae in 3D  
ApJL 2014, 785, L29

Authors observe no runaway explosion by the end of the full 3D simulation at 185 ms after bounce.

Detailed 3D MR supernova simulations are necessary.
3D tetrahedral grid

3D – work in progress
Conclusions

- Magnetorotational mechanism (MRM) produces enough energy for the core collapse supernova.
- The MRM is weakly sensitive to the neutrino cooling mechanism and details of the EoS.
- MR supernova shape depends on the configuration of the magnetic field and is always asymmetrical.
- MRDI is developed in MR supernova explosion.