

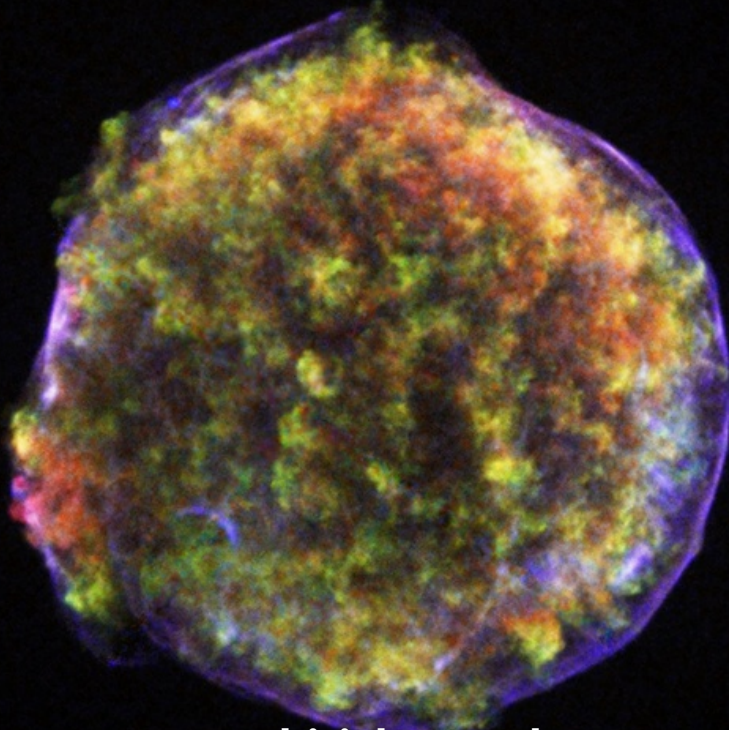
Kinetic Plasma Processes in Diffusive Shock Acceleration

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Tycho's SNR

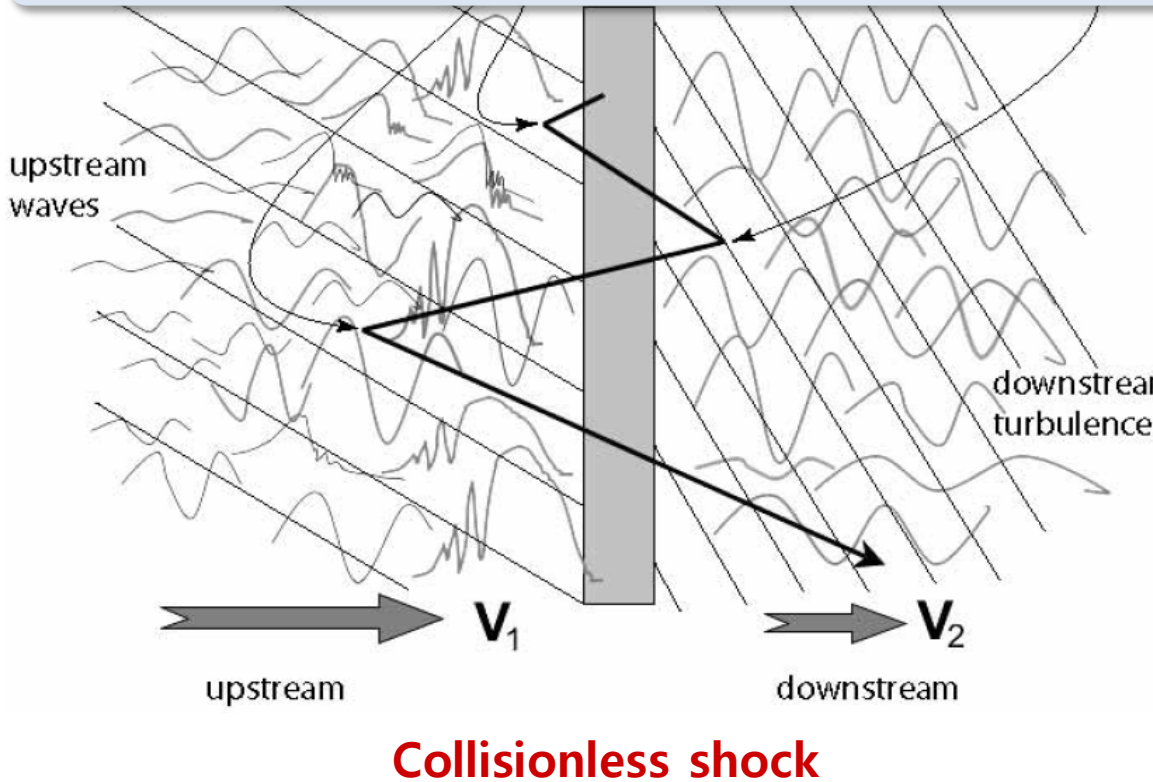
SN1006



whitish purple: X-ray synchrotron from CR electrons

2014-10-10

H.E. Particles are accelerated at shocks by Fermi 1st order process



test - particle solution

$$f_{\text{test}}(p) \propto p^{-q_{\text{test}}}$$

$$q_{\text{test}} = \frac{3u_1}{(u_1 - u_2)}$$

→ 4 for large M
(strong shocks)

$$\Rightarrow N(E) \propto E^{-2}$$

MHD waves in a converging flow act as converging mirrors

→ particles are scattered by MHD waves and isotropized in local fluid frame

→ cross the shock many times, gain $\frac{\Delta p}{p} \sim \frac{u_1 - u_2}{v}$ at each shock crossing

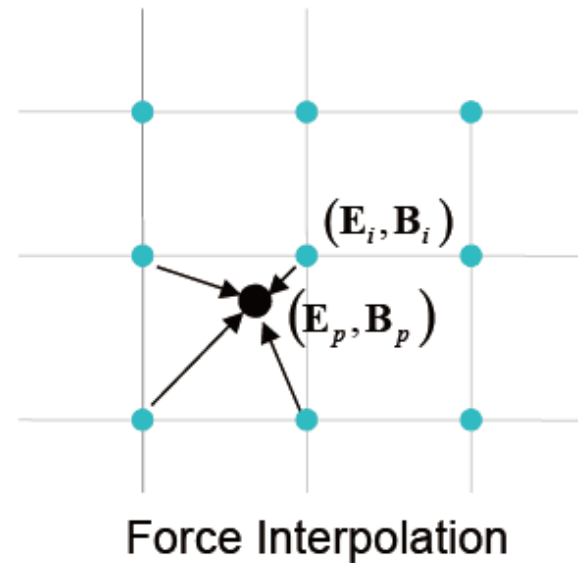
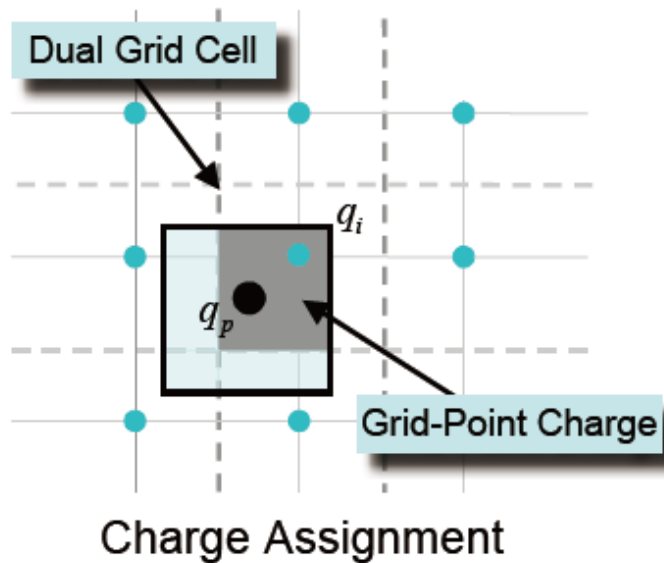
Kinetic Plasma Simulations: PIC (Particle In Cell)/ Hybrid

PIC Approach to Vlasov Equation

(Slide from Spitkovsky)

- Lorentz-Force: $\mathbf{F}_p = q\mathbf{E}_p + \frac{q}{m}(\mathbf{p}_p \times \mathbf{B}_p)$
- Solve Maxwell Equations on grid
- “Grid aliasing” (Birdsall et al.)

-can follow kinetic plasma processes:
e.g. wave-particle interactions
-provide the most complete pictures,
but very expensive



PIC (Amano & Hoshino 2012; Riquelme & Spitkovsky 2011, Guo + 2014, Kato 2014)

Hybrid (Giacalone 2013; Gargate & Spitkovsky 2011, Caprioli & Spitkovsky 2014): **Ion PIC + Electron fluid**

Collisionless Boltzman Eqn (Vlaso - Maxwell): $f(\vec{x}, \vec{p}, t)$ in 6D phase space

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla} f + \vec{F} \cdot \frac{\partial f}{\partial \vec{p}} = 0 \quad \left(\text{where } \vec{F} = q[\vec{E} + \frac{1}{c}(\vec{v} \times \vec{B})] \right)$$

Fokker - Planck Eqn for collisionless plasma in quasi - linear limit :

gyro - phase averaged $f(x, p, \mu, t)$ in 3D phase space

$$\begin{aligned} \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} = & \frac{\partial}{\partial \mu} \left(D_{\mu\mu} \frac{\partial f}{\partial \mu} \right) + \frac{\partial}{\partial \mu} \left(D_{\mu p} \frac{\partial f}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{p\mu} \frac{\partial f}{\partial \mu} \right) \\ & + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p} \right) \end{aligned}$$

e.g. Schlickeiser 1989

where $D_{\mu\mu}, D_{\mu p}, D_{p\mu}, D_{pp}$: wave - particle interactions due to turbulence

Diffusion - Convection Eqn (with Alfvén waves)

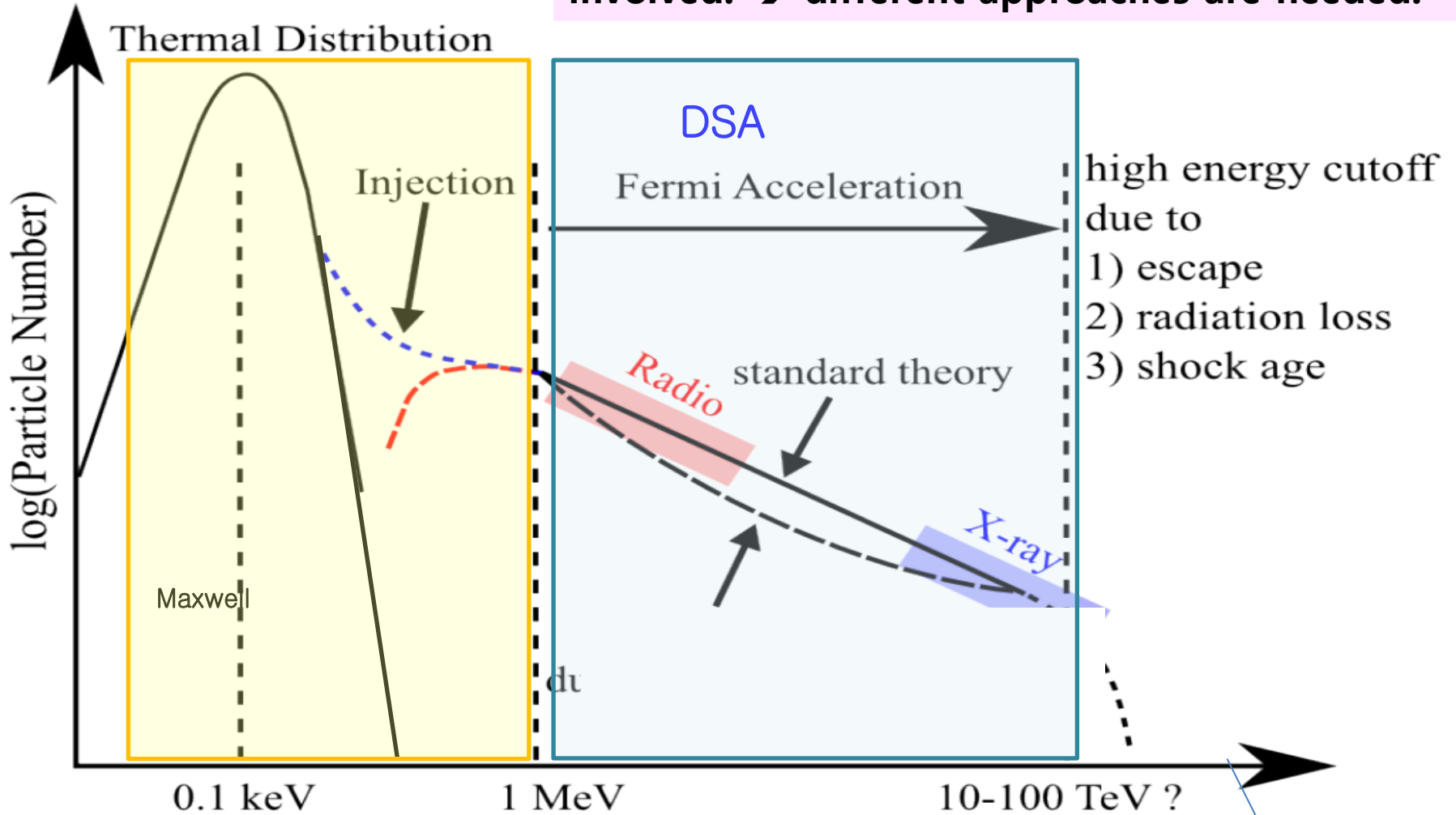
pitch - angle averaged $f(x, p, t)$: isotropic part

e.g. Skilling 1975

$$\frac{\partial f}{\partial t} + (u + u_w) \frac{\partial f}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} (u + u_w) \cdot p \frac{\partial f}{\partial p} + \frac{\partial}{\partial x} \left(\kappa \frac{\partial f}{\partial x} \right) + \frac{1}{p} \frac{\partial^2 f}{\partial p^2} \approx \text{2nd order: small in DSA}$$

$f(x, p, t)$: isotropic part

Wide ranges of space, time, energy scales are involved. → different approaches are needed.



PIC/Hybrid simulations
wave-particle interactions
Injection, pre-heating via
plasma instabilities

Diffusion Convection Eqn
phenomenological models
for injection, diffusion, wave
drifts...

from Amano
2009

Time-dependent DSA simulations (Kang, Jones,..)

Diffusion Convection Equation with Phenomenological recipes

$$\frac{\partial f}{\partial t} + (u + u_w) \frac{\partial f}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} (u + u_w) \cdot p \frac{\partial f}{\partial p} + \frac{\partial}{\partial x} [\kappa(x, p) \frac{\partial f}{\partial x}] + Q(x, p)$$

$u_w \approx$ wave drift speed $\approx V_A(x) = B(x) / \sqrt{4\pi\rho}$: MFA

$\kappa(x, p) \approx \kappa^* p \propto B(x)^{-1}$: Bohm - like diffusion

$Q(x, p) =$ injection of suprathermal ptls into Fermi process

$$\frac{\partial \rho}{\partial t} + \frac{\partial (u\rho)}{\partial x} = 0$$

ordinary gasdynamics EQs + P_c terms

(1D plane quasi-parallel shock)

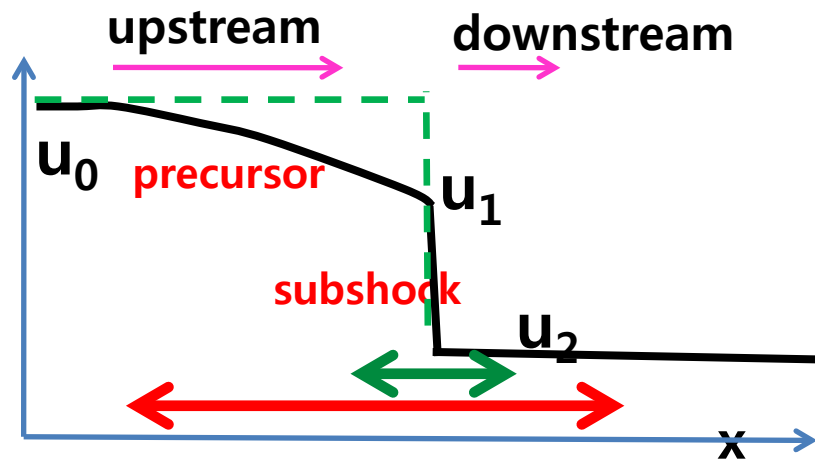
$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x} (\rho u^2 + P_g + P_c) = 0$$

$$\frac{\partial (\rho e_g)}{\partial t} + \frac{\partial}{\partial x} (\rho e_g u + P_g u) = -u \frac{\partial P_c}{\partial x} + W - L$$

$W =$ wave dissipation heating, $L =$ thermal energy loss due to injection

Nonlinear DSA effects: CR pressure feedback

CR modified shock with high Mach no.

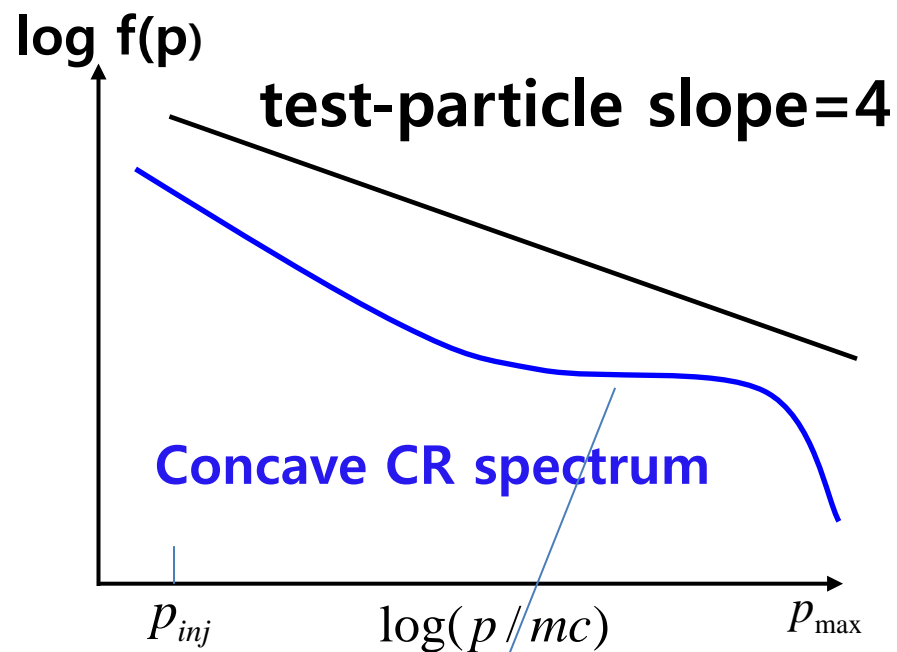


across the subshock

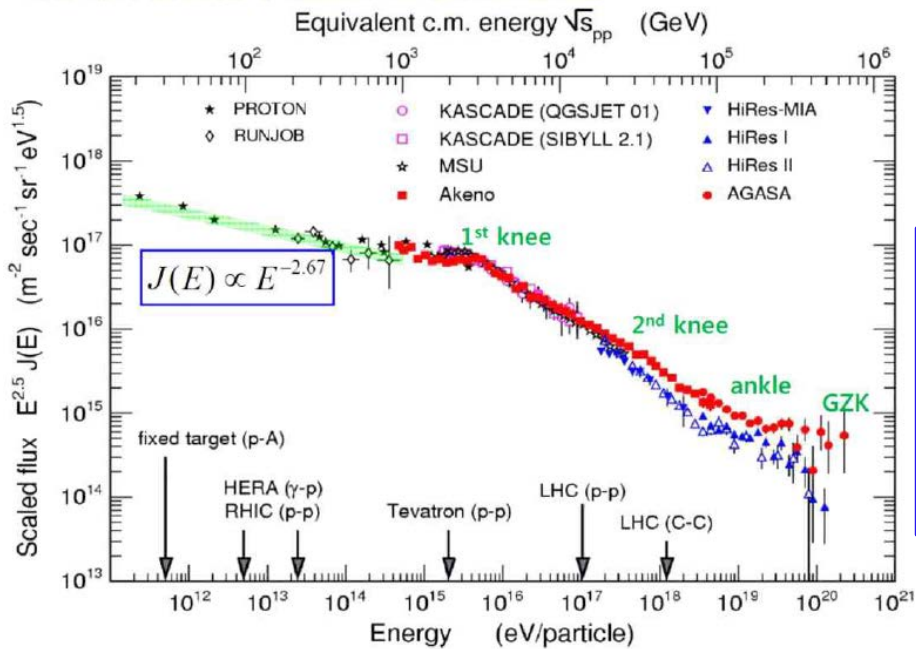
$$q_s = \frac{3 \cdot (u_1)}{u_1 - u_2}$$

across the total shock

$$q_t = \frac{3 \cdot (u_0)}{u_0 - u_2}$$



Flatter than test-particle power-law



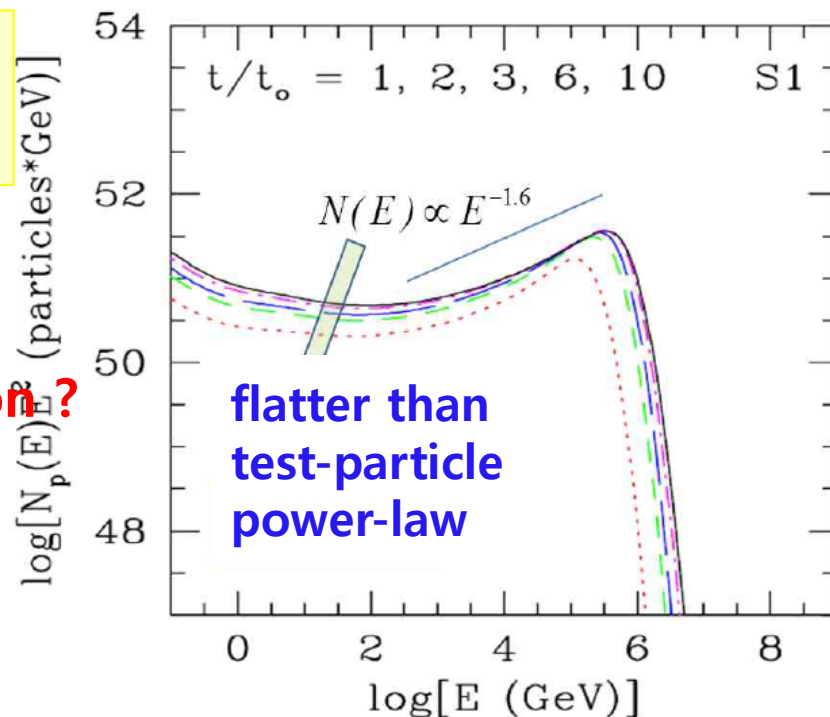
Observed CR spectrum at Earth: $J(E)$

$J(E) \propto E^{-2.7}$ below the knee
 if mean propagation length: $\Lambda \propto E^{0.6}$
 $\Rightarrow N(E) \propto E^{-2.2}$ at sources

**Nonlinear DSA simulations
 → CR spectrum at source: $N(E)$**

$N(E) \propto E^{-1.6}$ at SNR

Nonlinear DSA simulation of SNR



Discrepancy bwn theory and observation?

→ possible solutions

- 1) MFA and
- 2) Alfvénic drift:

Wave-Particle Interactions: CR streaming Instabilities

streaming CR protons upstream of parallel shocks

- excite resonant Alfvén waves (i.e. ion/ion streaming instability)
- amplify B field (Bell 1978, Lucek & Bell 2000)

$$\lambda_w \sim r_g(p)$$

resonant waves

$$\lambda_w \ll r_g(p)$$

nonresonant waves

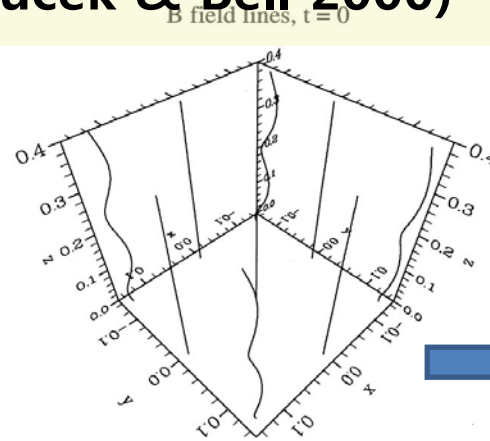
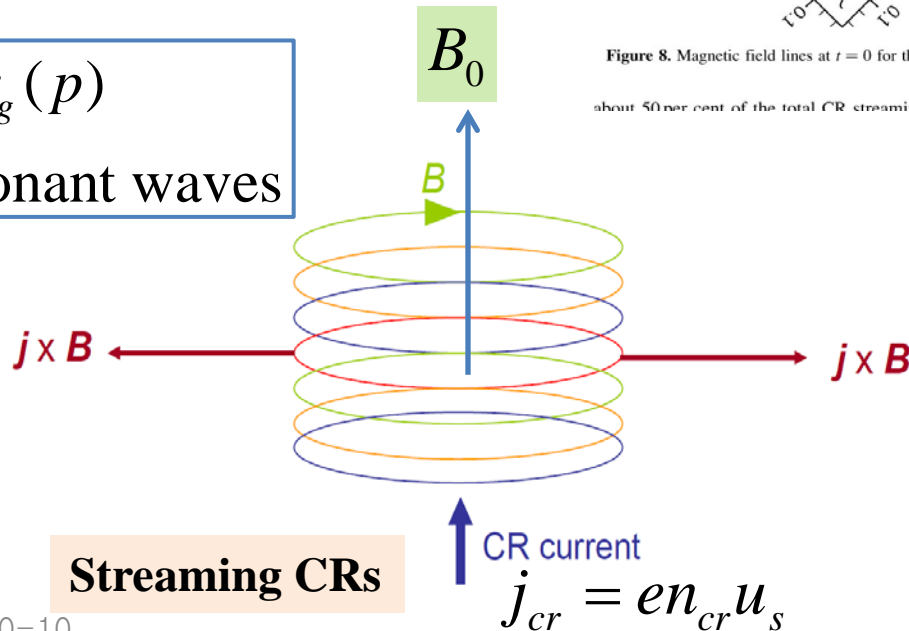


Figure 8. Magnetic field lines at $t = 0$ for the three-dimensional run.

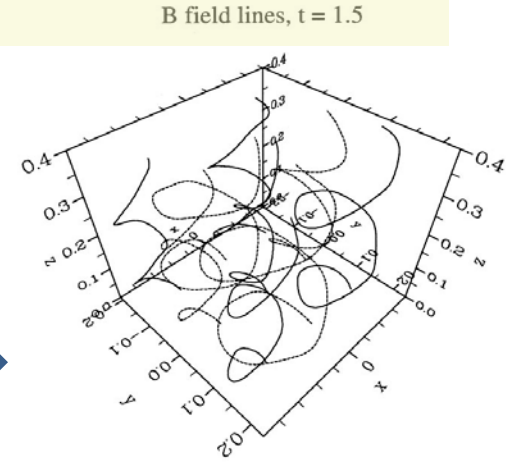


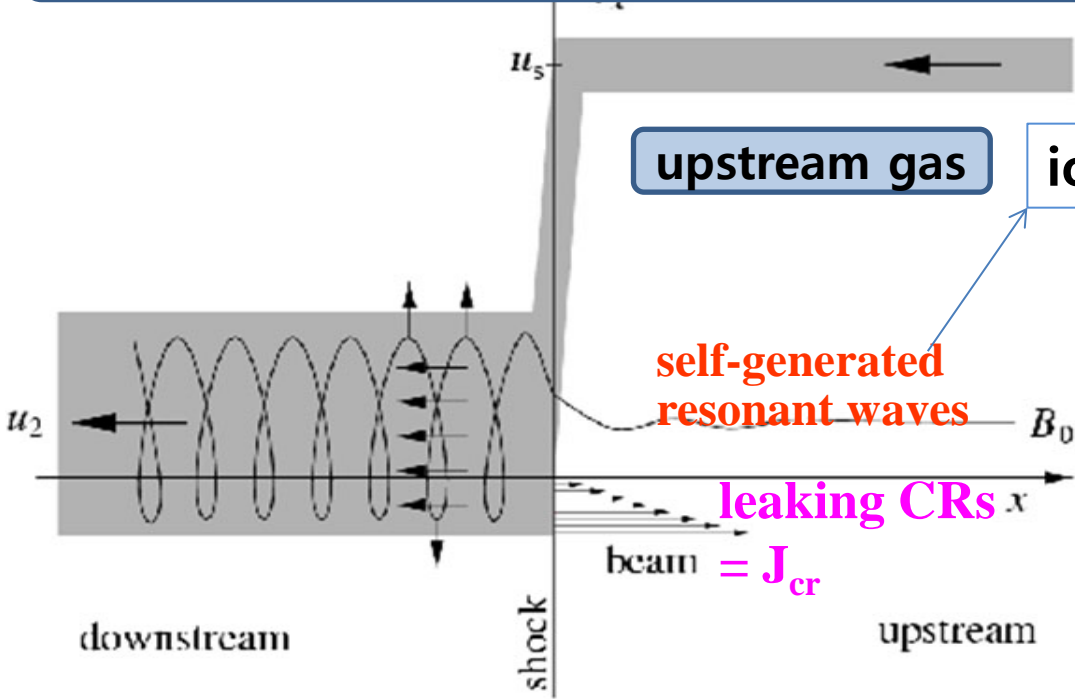
Figure 9. Magnetic field lines after 1.5 CR gyrations for the three-dimensional run.

Cosmic-ray current drives nonresonant instability by stretching field lines (Bell, 2004)

$$\frac{B_{\perp}}{B_0} \sim \frac{B_{\parallel}}{B_0} \sim 10$$

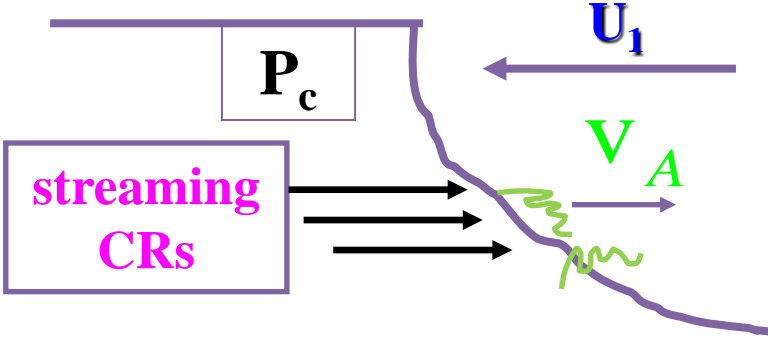
Riquelme & Spitkovsky 2009⁹

Alfvénic Drift: leads to steepening of N(E)



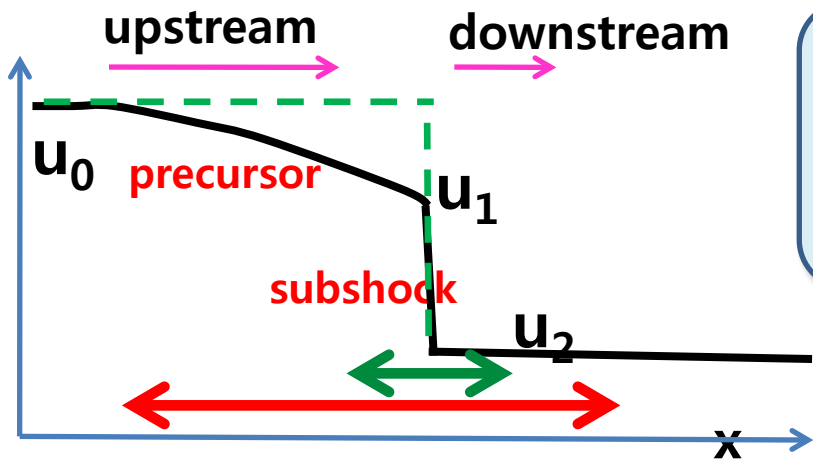
$$V_A(x) \sim \frac{B(x)}{\sqrt{4\pi\rho}} \text{ on amplified field}$$

- Faster Alfvénic drift
- Softer CR spectrum



- waves are generated by streaming instability.
- waves drift upstream with $u_w \approx V_A$
- CRs are scattered and isotropized in the wave frame (instead of fluid frame)
- Experience smaller $\Delta u \Rightarrow u - V_A$
- less efficient acceleration
- Steepening of CR spectrum

Bell 1978



CR modified shock
 + Magnetic Field Amplification
 + fast Alfvénic Drift

across the subshock

$$q_s = \frac{3 \cdot (u_1 - V_A)}{u_1 - V_A - u_2}$$

across the total shock

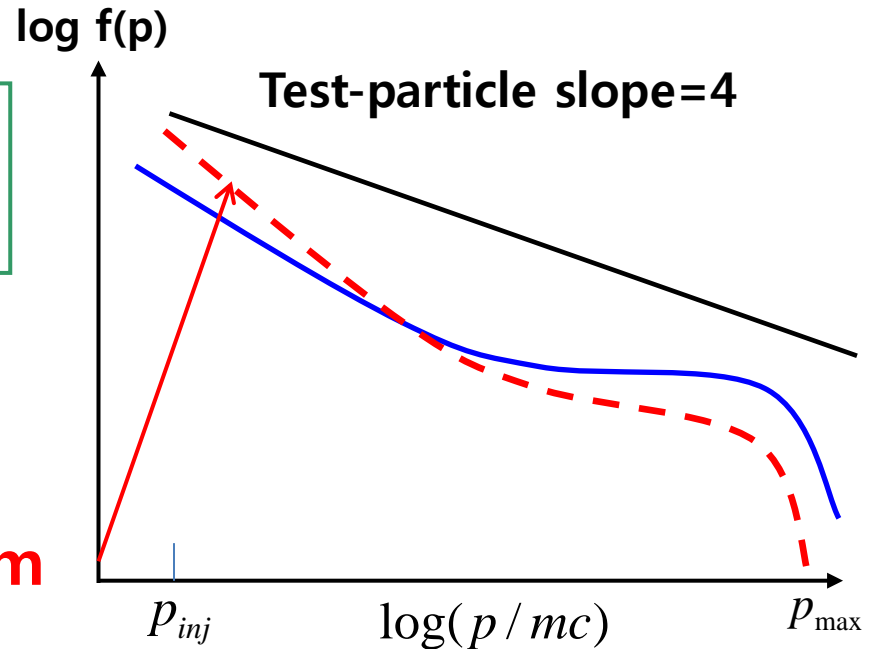
$$q_t = \frac{3 \cdot (u_0 - V_A)}{u_0 - V_A - u_2}$$

Alfvénic Drift → softer spectrum

higher CR modification ⇒ MFA

⇒ larger B_1 & B_2 ⇒ larger V_A

⇒ steeper slope ⇒ less efficient acceleration



Highly nonlinear problem !

DSA simulations of SNRs with **MFA & Alfvenic Drift**

in a **co-expanding** frame which expands with the forward shock.

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{a} \frac{\partial (v \tilde{\rho})}{\partial x} = -\frac{2}{ax} \tilde{\rho} v$$

ordinary gasdynamics EQs + P_c terms

$$\frac{\partial (\tilde{\rho} v)}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} v^2 + \tilde{P}_g + \tilde{P}_c)}{\partial x} = -\frac{2}{ax} \tilde{\rho} v^2 - \frac{\dot{a}}{a} \tilde{\rho} v - \ddot{a} x \tilde{\rho}$$

$$\begin{aligned} \frac{\partial (\tilde{\rho} \tilde{e}_g)}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} \tilde{e}_g v + \tilde{P}_g v + \tilde{P}_c v)}{\partial x} = & -\frac{v}{a} \frac{\partial \tilde{P}_c}{\partial x} - \frac{2}{ax} (\tilde{\rho} \tilde{e}_g v + \tilde{P}_g v) \\ & - 2 \frac{\dot{a}}{a} \tilde{\rho} \tilde{e}_g - \ddot{a} x \tilde{\rho} v - \tilde{L}(x, t) \end{aligned}$$

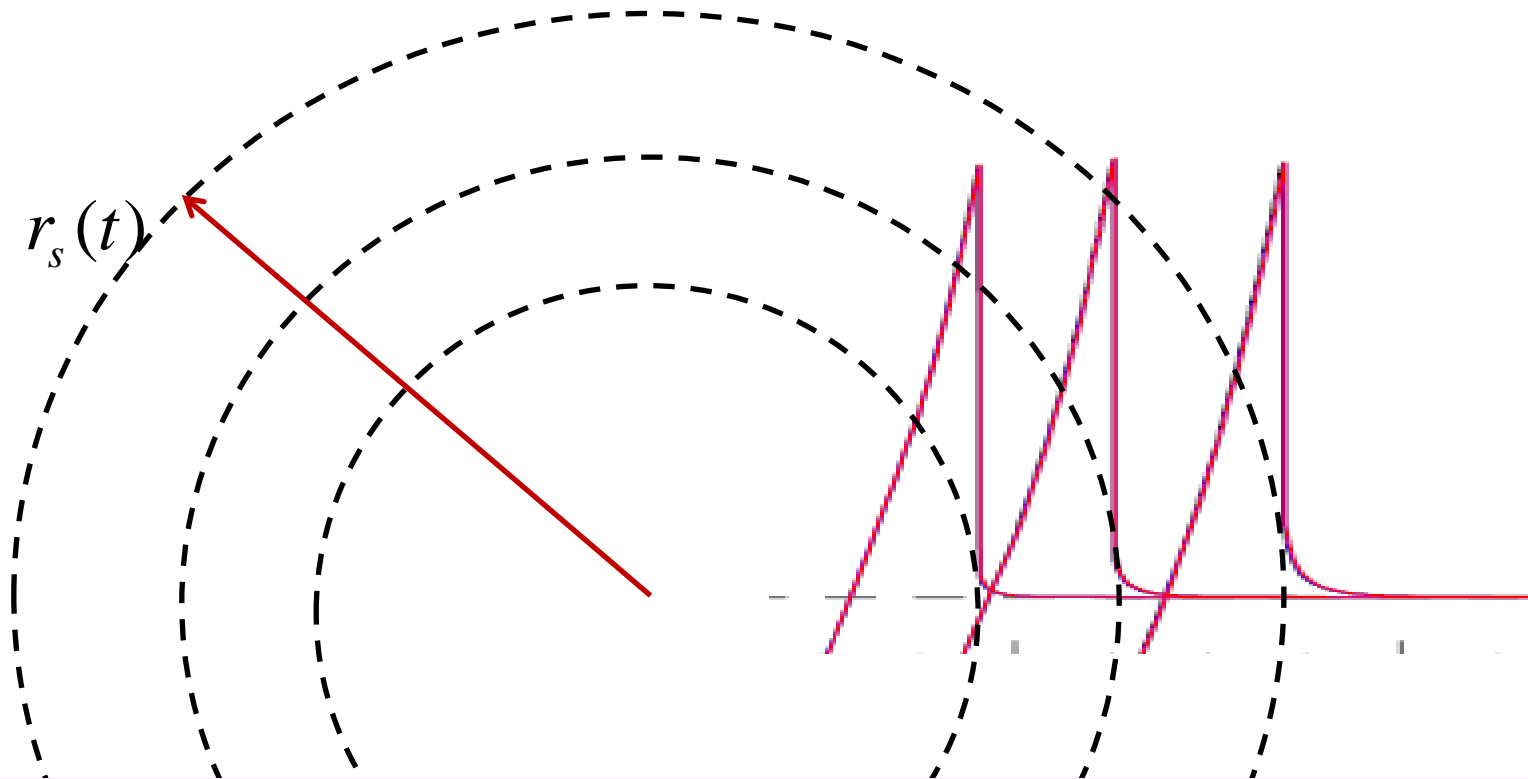
Diffusion Convection Equation for $g = f(r, p, t) p^4$ **for protons and electrons**

$$\frac{\partial \tilde{g}}{\partial t} + \frac{(v - u_w)}{a} \frac{\partial \tilde{g}}{\partial x} = \left[\frac{1}{3ax} \frac{\partial}{\partial x} (x^2 (v - u_w)) + \frac{\dot{a}}{a} \right] \left(\frac{\partial \tilde{g}}{\partial y} - 4 \tilde{g} \right) + 3 \frac{\dot{a}}{a} \tilde{g} + \frac{1}{a^2 x^2} \frac{\partial}{\partial x} \left(x^2 \kappa \frac{\partial \tilde{g}}{\partial x} \right)$$

$x = r / a$: co - expanding coordinate, $a =$ expansion factor, $y = \ln p$

CRASH code in 1D spherical geometry: Kang et al. 2013

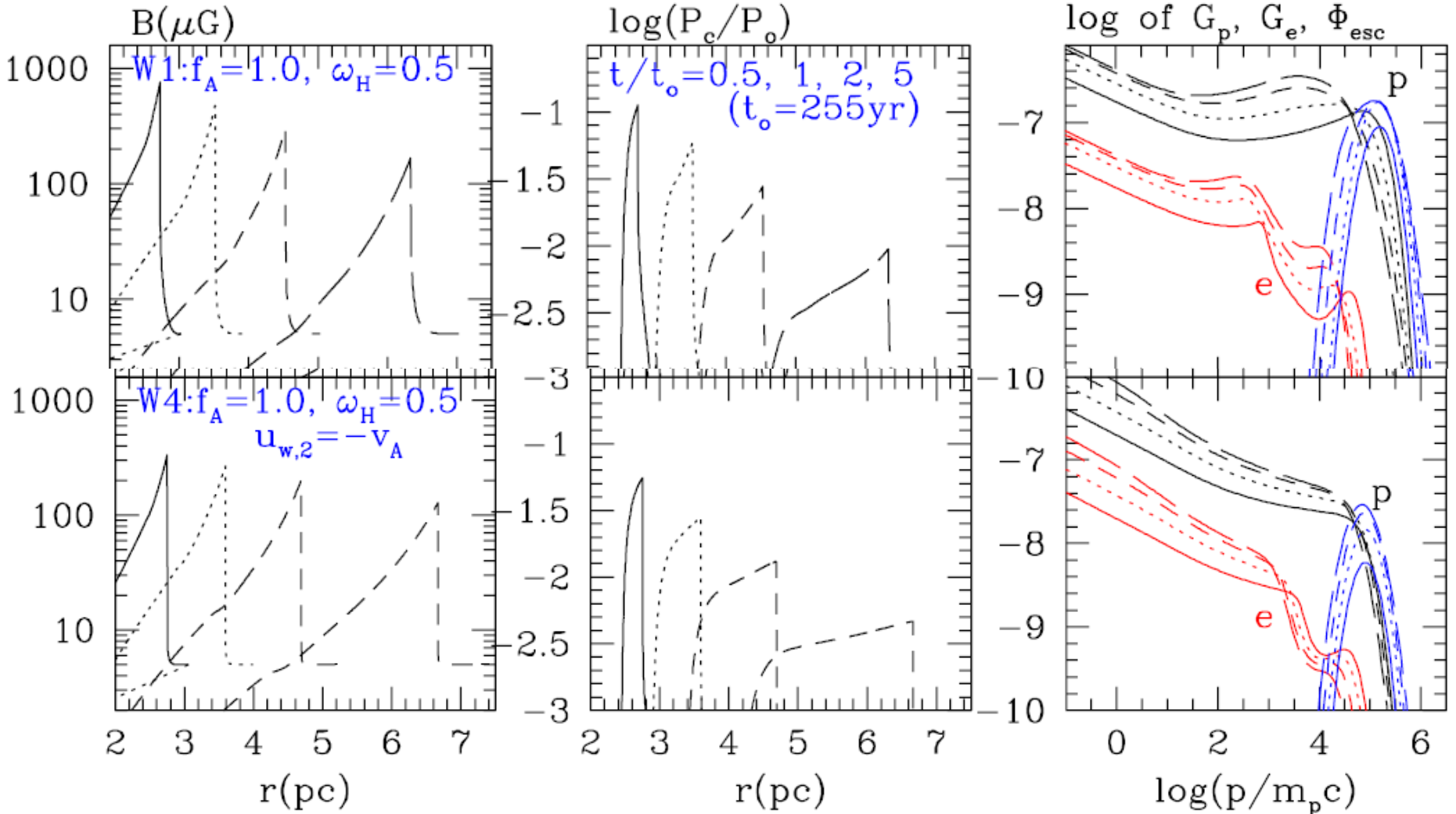
The expansion factor, $a(t)$, is defined so that the shock position, $x_s = \text{constant}$, in co-expanding coordinate, while $r_s(t) \equiv a(t) \cdot x_s$ expands in physical coordinate



The shock position is fixed in the co-expanding frame.

Type Ia SNR Model: 1D spherical CRASH

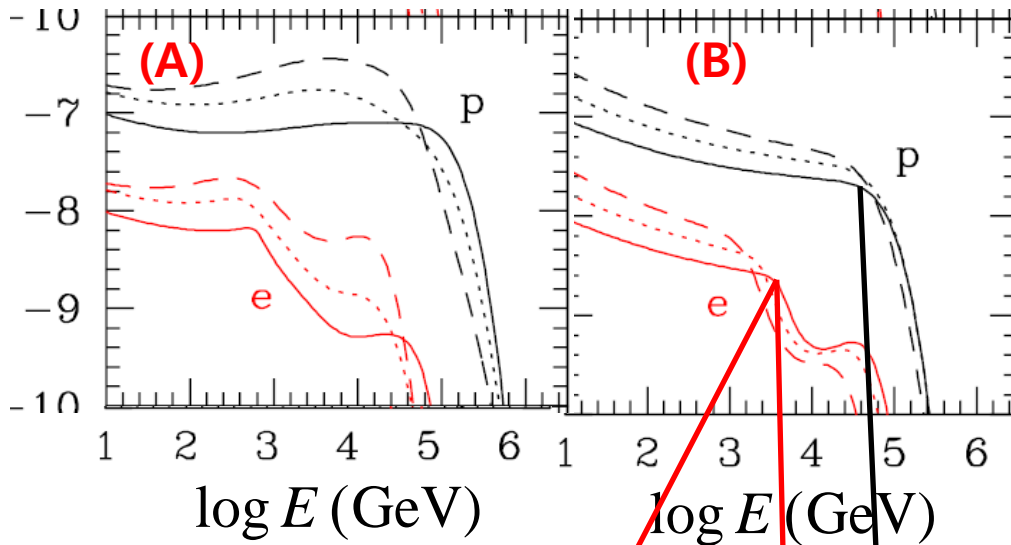
$$M_{ej} = 1.4 M_{\odot}, \quad E_o = 10^{51} \text{ ergs}, \quad n_{\text{ISM}} = 0.3 \text{ cm}^{-3}, \quad T_0 = 3 \times 10^4 \text{ K}, \quad B_0 = 5 \mu\text{G}$$



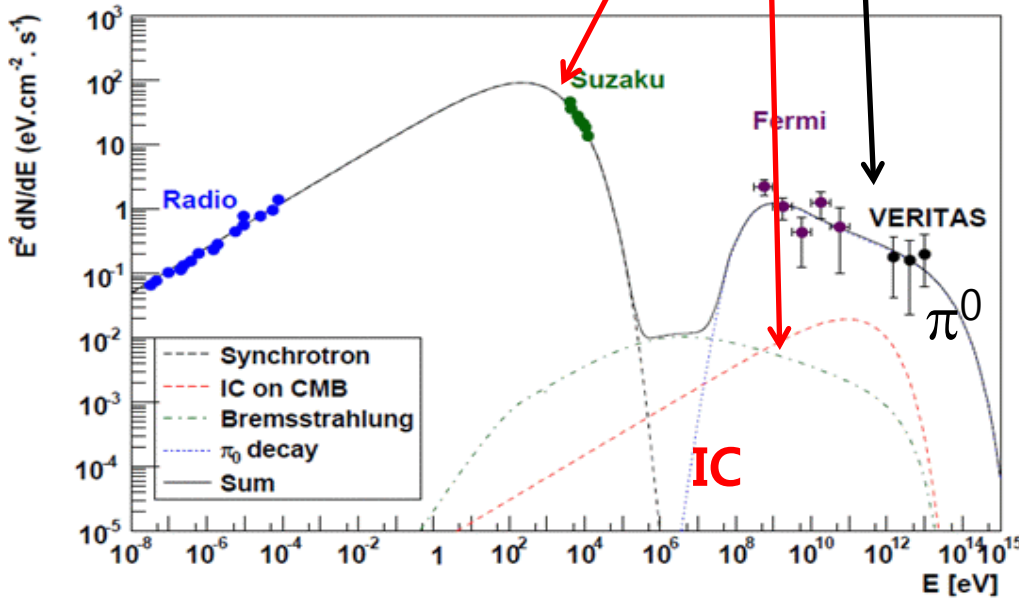
Two different models for $B(x)$ and AD

$$G(p) = \int_{r_i}^{r_s} 4\pi \cdot f(p) p^4 \cdot r^2 dr$$

$E^2 N(E)$: volume integrated CR spectrum depends on $B(r)$ model.



time - dependent evolution of $B(r, t) \Rightarrow V_A(r, t) \& \kappa(p, r, t)$
 $\Rightarrow q$: slope
 $\& E_{\max, p}, E_{\max, e}$ (cooling)

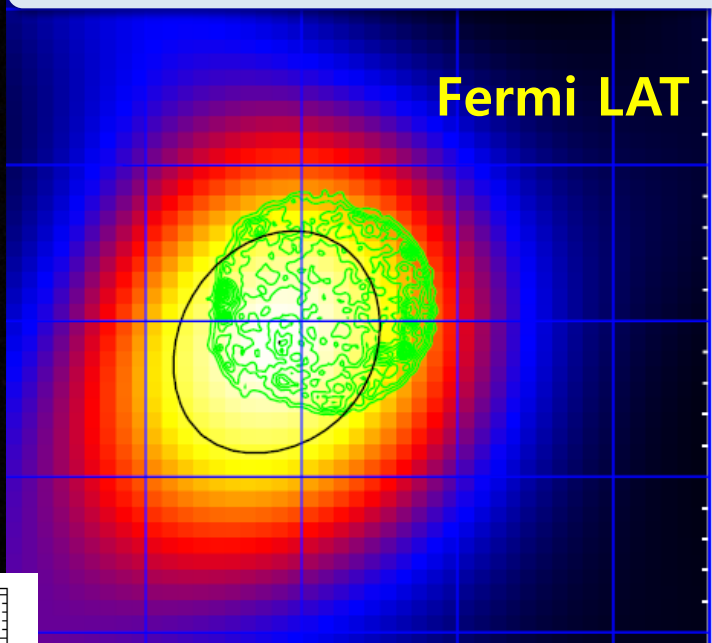
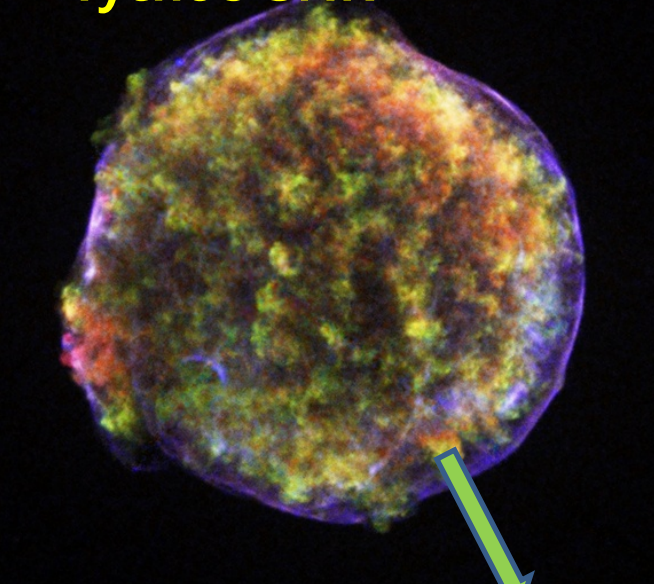


So details of DSA modeling are important in predicting nonthermal emission from SNRs.

Highest energy end of CR spectra determines keV, GeV-TeV emission.

Tycho's SNR

Nonthermal emission from Tycho's SNRs

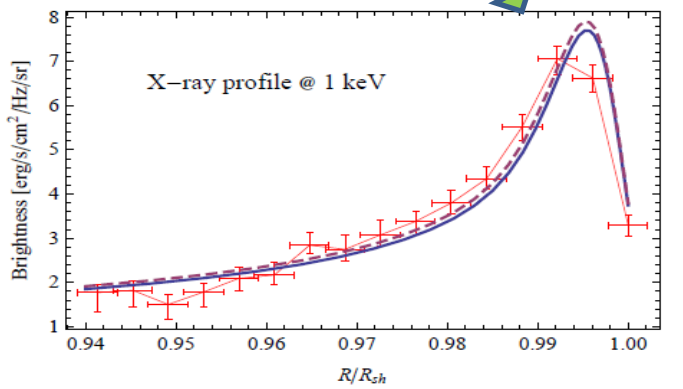


Proton spectrum

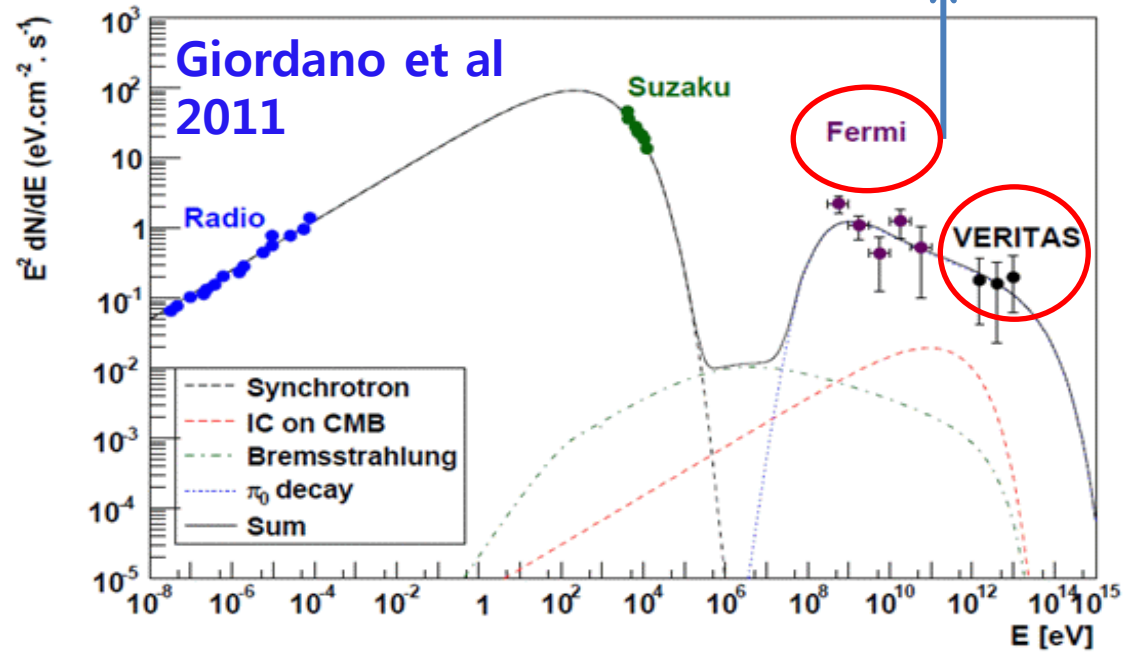
$$N_p(E) \propto E^{-2.3}$$

$$B_2 \sim 200 \mu\text{G}$$

π^0 decay emission



Projected X-ray emission
 → $B_2 = 100\text{-}500 \mu\text{G}$
 → **Magnetic Field Amplification at shocks**



Summary: Take-home messages

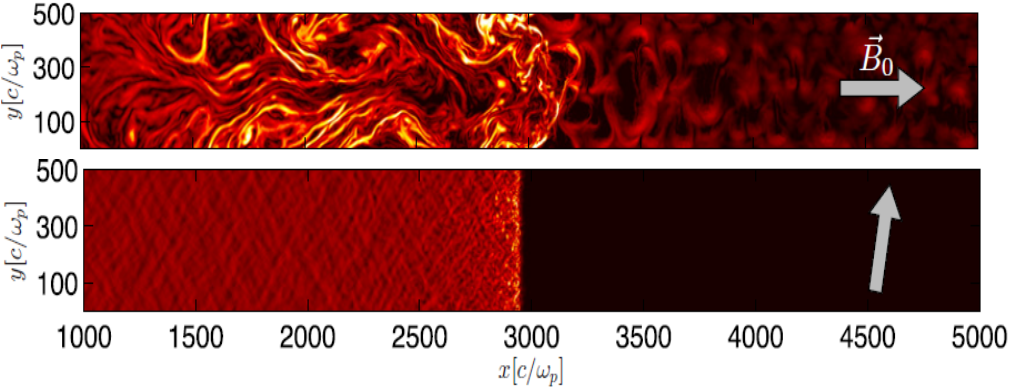
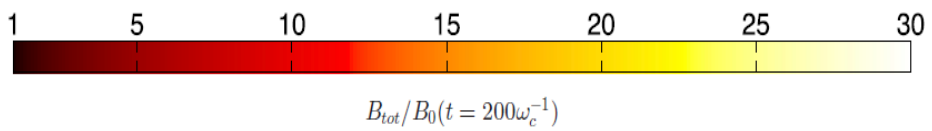
**-Wave-particle interactions are important in DSA theory:
So Various plasma instabilities, MFA, Alfvenic drift, pre-heating of protons/electrons should be studied further by plasma simulations.**

→ provide phenomenological models for DSA simulations

-Detailed plasma physics of DSA are important in predicting nonthermal emission from SNRs.

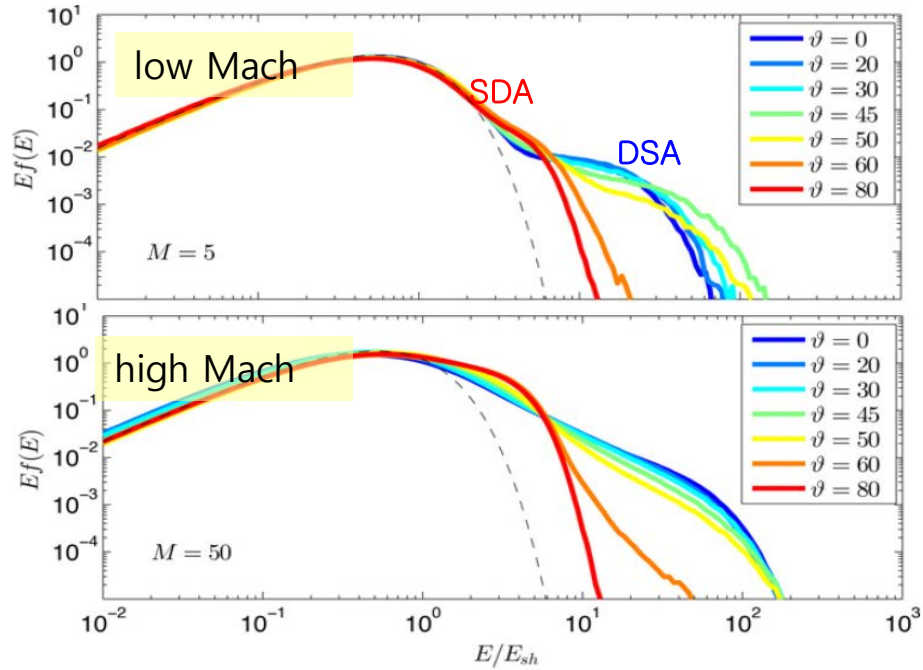
→ Testing “SNR hypothesis for the origin of Galactic CRs”

Proton acceleration at shocks: 2D Hybrid simulation



At parallel shocks, stream of accelerated protons into upstream
 → self-generated waves
 → Turbulent B amplification

At perpendicular shocks
 No accelerated protons into upstream
 → No turbulent waves



DSA = Diffusive Shock Acceleration at quasi-parallel shocks ($\theta_B < 45^\circ$)

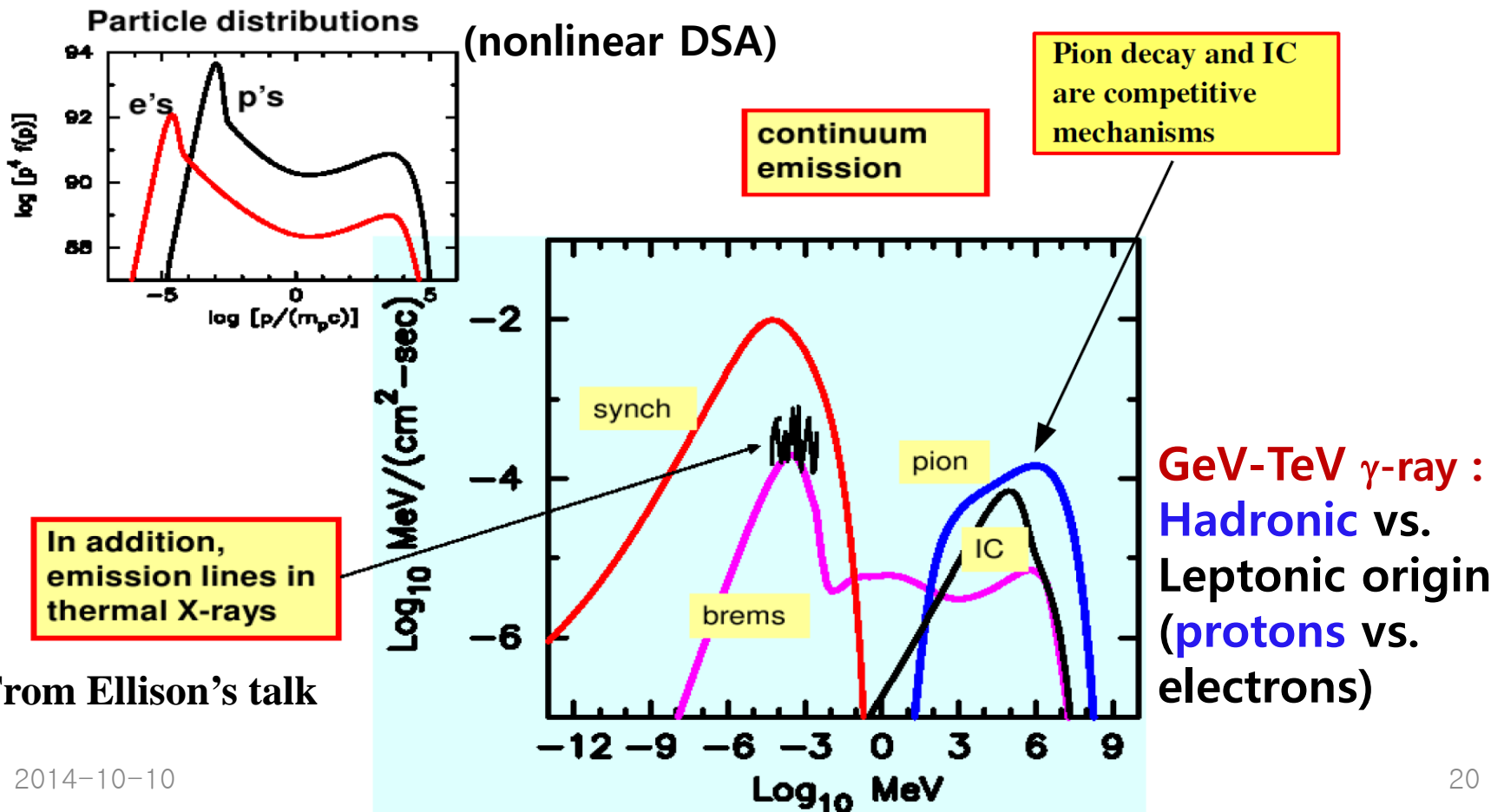
SDA = Shock Drift Acceleration at quasi-perpendicular shocks ($\theta_B > 45^\circ$)

At quasi-parallel shocks
 -CR acceleration efficiency ~10-20% in energy
 -CR injection efficiency ~ 10^{-3} in number

Nonthermal radiation from CRs accelerated at SNR shocks

→ provide observational evidence and constraints for CR acceleration.

- CR e + B field → Synchrotron (radio – X-ray)
- thermal & non-thermal bremsstrahlung
- CR e + CMBR → Inverse Compton scattering → TeV γ -ray
- CR p + p → π^0 decay → 100 GeV γ -ray



Numerical Methods to study Particle Acceleration

*Monte Carlo Simulations with a scattering model:

- scattered with a prescribed scattering model,
- assume a **steady-state shock structure with FEB**

e.g. Ellison, Baring, Jones, Vladimirov +

$$\lambda \propto \frac{R^\alpha}{\rho} = \lambda_0 \left(\frac{A}{Q} \right)^\alpha \left(\frac{v}{u_2} \right)^\alpha \left[\frac{\rho_2}{\rho(x)} \right],$$

*Semi-analytic approach

- analytical solution of the **stationary diffusion-convection equation**
+ gasdynamic conservation equations

e.g. Blasi, Amato, Caprioli, Molino +

*Time-dependent diffusion-convection Simulations

- **diffusion approximation** based on isotropy of particle distribution
- follow time dependent evolution of $f(x,p,t)$ + gasdynamics Eqs

e.g. Berezhko et al., Kang & Jones

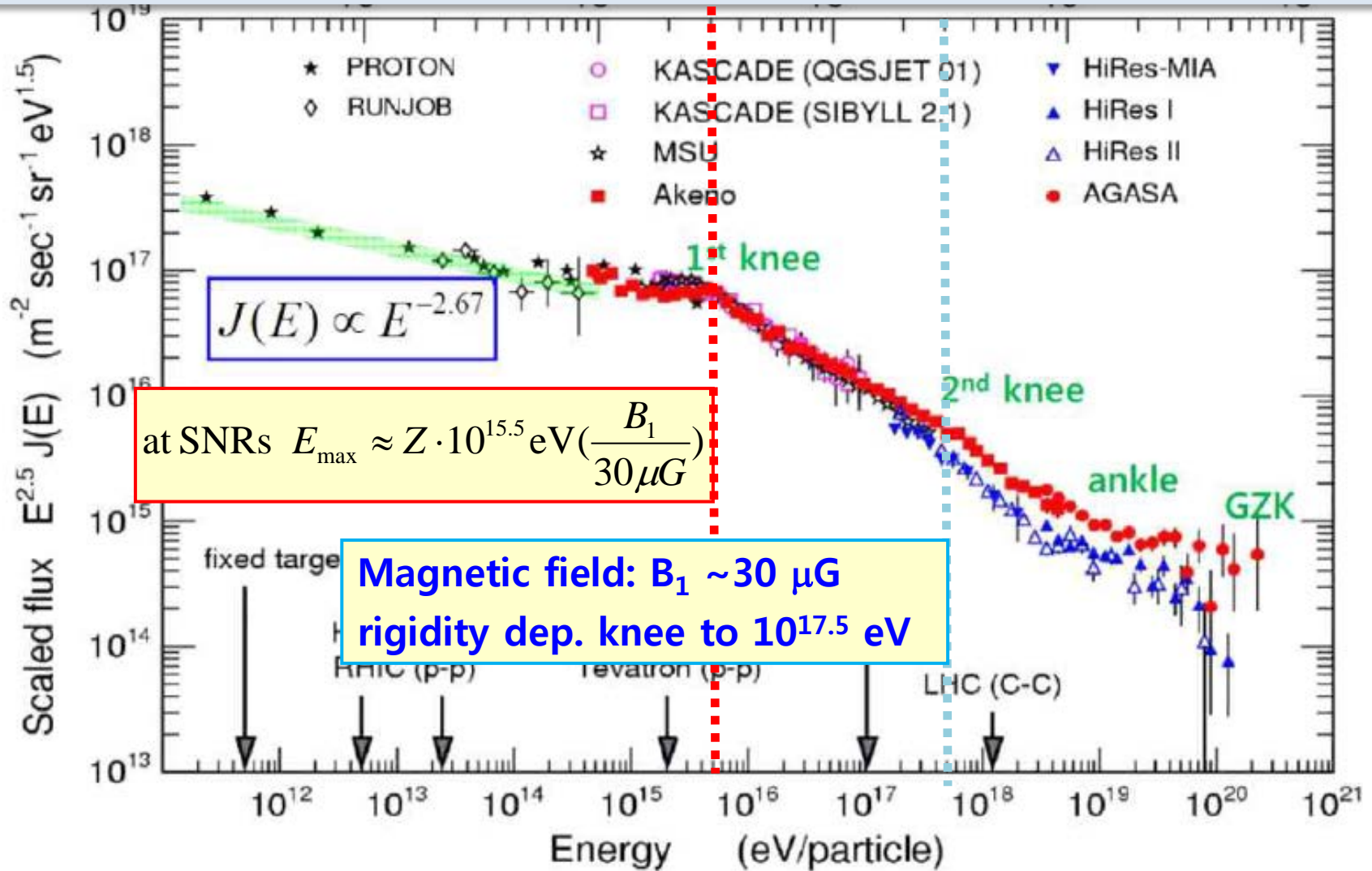
*Full PIC & Hybrid plasma simulations: 3D is required, non-relativistic

- follow individual particles and magnetic fields
- provide the most complete pictures, but very expensive

PIC (Amano & Hoshino 2012; Riquelme & Spitkovsky 2011, Guo + 2014, Kato 2014)

hybrid (Giacalone 2013; Gargate & Spitkovsky 2011, Caprioli & Spitkovsky 2014)

Energy Spectrum of Cosmic Rays are power-law of $\sim E^{-3}$



Below $\sim 10^{17.5} \text{ eV}$: Galactic CRs are nuclei accelerated in SNRs
if magnetic fields are amplified to $B_1 \sim 30 \mu\text{G}$ at the shock.