Kinetic Plasma Processes in Diffusive Shock Acceleration

Hyesung Kang
Pusan National University, Pusan, Korea

Tycho’s SNR

whitish purple: X-ray synchrotron from CR electrons

SN1006
H.E. Particles are accelerated at shocks by Fermi 1\textsuperscript{st} order process

\[ f_{\text{test}}(p) \propto p^{-q_{\text{test}}} \]

\[ q_{\text{test}} = \frac{3u_1}{(u_1 - u_2)} \]

\[ \Rightarrow 4 \text{ for large } M \]

(\textit{strong shocks})

\[ N(E) \propto E^{-2} \]

\textbf{Collisionless shock}

\textbf{MHD waves in a converging flow act as converging mirrors}

\rightarrow \text{ particles are scattered by MHD waves and isotropized in local fluid frame}

\rightarrow \text{ cross the shock many times, gain } \frac{\Delta p}{p} \sim \frac{u_1 - u_2}{v} \text{ at each shock crossing}
Kinetic Plasma Simulations: PIC (Particle In Cell)/ Hybrid

PIC Approach to Vlasov Equation

- Lorentz-Force: \( F_p = qE_p + \frac{q}{m} \left( p_p \times B_p \right) \)
- Solve Maxwell Equations on grid
- “Grid aliasing” (Birdsall et al.)

(Slide from Spitkovsky)

- can follow kinetic plasma processes: e.g. wave-particle interactions
- provide the most complete pictures, but very expensive

Dual Grid Cell

Grid-Point Charge

Charge Assignment

Force Interpolation


Hybrid (Giacalone 2013; Gargate & Spitkovsky 2011, Caprioli & Spitkovsky 2014): Ion PIC + Electron fluid
Collisionless Boltzmann Eqn (Vlaso - Maxwell): $f(\tilde{x}, \tilde{p}, t)$ in 6D phase space

$$\frac{\partial f}{\partial t} + \frac{\tilde{p}}{m} \cdot \nabla f + \tilde{F} \cdot \frac{\partial f}{\partial \tilde{p}} = 0 \quad (\text{where } \tilde{F} = q[E + \frac{1}{c}(\tilde{v} \times \tilde{B})])$$

Fokker - Planck Eqn for collisionless plasma in quasi-linear limit:
gyro-phase averaged $f(x, p, \mu, t)$ in 3D phase space

$$\frac{\partial f}{\partial t} + v \cdot \frac{\partial f}{\partial x} = \frac{\partial}{\partial \mu} \left(D_{\mu\mu} \frac{\partial f}{\partial \mu}\right) + \frac{\partial}{\partial \mu} \left(D_{\mu p} \frac{\partial f}{\partial p}\right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial \mu}\right)$$

$$+ \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p}\right)$$

e.g. Schlickeiser 1989

where $D_{\mu\mu}, D_{\mu p}, D_{p\mu}, D_{pp}$ : wave-particle interactions due to turbulence

Diffusion - Convection Eqn (with Alfven waves)
pitch-angle averaged $f(x, p, t)$: isotropic part

$$\frac{\partial f}{\partial t} + (u + u_w) \frac{\partial f}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} (u + u_w) \cdot p \frac{\partial f}{\partial p} + \frac{\partial}{\partial x} (\kappa \frac{\partial f}{\partial x}) + \frac{1}{p} \frac{\partial}{\partial p} \frac{\partial f}{\partial p}$$

e.g. Skilling 1975

2nd order: small in DSA
Wide ranges of space, time, energy scales are involved. ➔ different approaches are needed.

\[ f(x, p, t) : \text{isotropic part} \]

**Thermal Distribution**

**Injection**

**Diffusion Convection Eqn**

**Phenomenological models**

**for injection, diffusion, wave drifts...**

**PIC/Hybrid simulations**

**Wave-particle interactions**

**Injection, pre-heating via plasma instabilities**

**DSA**

**Fermi Acceleration**

- high energy cutoff due to
  1) escape
  2) radiation loss
  3) shock age

**from Amano 2009**
Diffusion Convection Equation with Phenomenological recipes

\[
\frac{\partial f}{\partial t} + (u + u_w) \frac{\partial f}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} (u + u_w) \cdot p \frac{\partial f}{\partial p} + \frac{\partial}{\partial x} [\kappa(x, p) \frac{\partial f}{\partial x}] + Q(x, p)
\]

\[u_w \approx \text{wave drift speed} \approx V_A(x) = \frac{B(x)}{\sqrt{4\pi \rho}}: \text{MFA}\]

\[\kappa(x, p) \approx \kappa^* p \propto B(x)^{-1}: \text{Bohm-like diffusion}\]

\[Q(x, p) = \text{injection of suprathermal pts into Fermi process}\]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (u \rho)}{\partial x} = 0
\]

\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x} (\rho u^2 + P_g + P_c) = 0
\]

\[
\frac{\partial (\rho e_g)}{\partial t} + \frac{\partial}{\partial x} (\rho e_g u + P_g u) = -u \frac{\partial P_c}{\partial x} + W - L
\]

\[W = \text{wave dissipation heating}, L = \text{thermal energy loss due to injection}\]
Nonlinear DSA effects: CR pressure feedback
CR modified shock with high Mach no.

\[ q_s = \frac{3 \cdot (u_1)}{u_1 - u_2} \]

\[ q_t = \frac{3 \cdot (u_0)}{u_0 - u_2} \]

across the subshock
across the total shock

Concave CR spectrum
Flatter than test-particle power-law
Nonliner DSA simulations \(\Rightarrow\) CR spectrum at source: \(N(E)\)

\[ N(E) \propto E^{-1.6} \text{ at SNR} \]

Discrepancy bwn theory and observation? \(\Rightarrow\) possible solutions

1) MFA and
2) Alfvenic drift:

\[ J(E) \propto E^{-2.7} \text{ below the knee} \]

if mean propagation length: \(\Lambda \propto E^{0.6}\)

\[ \Rightarrow N(E) \propto E^{-2.2} \text{ at sources} \]
Streaming CR protons upstream of parallel shocks → excite resonant Alfven waves (i.e. ion/ion streaming instability) → amplify B field (Bell 1978, Lucek & Bell 2000)

\[ \lambda_w \sim r_g(p) \] resonant waves

\[ \lambda_w \ll r_g(p) \] nonresonant waves

**Cosmic-ray current** drives nonresonant instability by stretching field lines (Bell, 2004)

\[ \frac{B_{\perp}}{B_0} \sim \frac{B_{\parallel}}{B_0} \sim 10 \]

Riquelme & Spitkovsky 2009
Alfvenic Drift: leads to steepening of $N(E)$

- waves are generated by streaming instability.
- waves drift upstream with $u_w \approx V_A$
- CRs are scattered and isotropized in the wave frame (instead of fluid frame)
  → Experience smaller $\Delta u$  $\Rightarrow u - V_A$
  → less efficient acceleration
  → Steepening of CR spectrum

$V_A (x) \sim \frac{B(x)}{\sqrt{4\pi \rho}}$ on amplified field

Faster Alfvenic drift
Softer CR spectrum

Bell 1978
CR modified shock
+ Magnetic Field Amplification
+ fast Alfvenic Drift

Alfvenic Drift $\Rightarrow$ softer spectrum

higher CR modification $\Rightarrow$ MFA
$\Rightarrow$ larger $B_1$ & $B_2$ $\Rightarrow$ larger $V_A$
$\Rightarrow$ steeper slope $\Rightarrow$ less efficient acceleration

$q_s = \frac{3 \cdot (u_1 - V_A)}{u_1 - V_A - u_2}$
$q_t = \frac{3 \cdot (u_0 - V_A)}{u_0 - V_A - u_2}$
in a **co-expanding** frame which expands with the forward shock.

\[
\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{a} \frac{\partial (\nu \tilde{\rho})}{\partial x} = - \frac{2}{ax} \tilde{\rho} \nu \\
\frac{\partial (\tilde{\rho} \nu)}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} \nu^2 + \tilde{P}_g + \tilde{P}_c)}{\partial x} = - \frac{2}{ax} \tilde{\rho} \nu^2 - \frac{\dot{\alpha}}{a} \tilde{\rho} \nu - \dot{\alpha} x \tilde{\rho} \\
\frac{\partial (\tilde{\rho} \tilde{e}_g)}{\partial t} + \frac{1}{a} \frac{\partial (\tilde{\rho} \tilde{e}_g \nu + \tilde{P}_g \nu + \tilde{P}_c \nu)}{\partial x} = \frac{\nu}{a} \frac{\partial \tilde{P}_c}{\partial x} - \frac{2}{ax} (\tilde{\rho} \tilde{e}_g \nu + \tilde{P}_g \nu) \\
- 2 \frac{\dot{\alpha}}{a} \tilde{\rho} \tilde{e}_g - \dot{\alpha} x \tilde{\rho} \nu - \tilde{L}(x,t)
\]

**Diffusion Convection Equation for** \( g = f(r, p, t) p^4 \) **for protons and electrons**

\[
\frac{\partial \tilde{g}}{\partial t} + \left( \frac{\nu - u_w}{a} \right) \frac{\partial \tilde{g}}{\partial x} = \left[ \frac{1}{3ax} \frac{\partial}{\partial x} (x^2 (\nu - u_w)) \right] + \frac{\dot{\alpha}}{a} \left( \frac{\partial \tilde{g}}{\partial y} - 4 \tilde{g} \right) + 3 \frac{\dot{\alpha}}{a} \tilde{g} + \frac{1}{a^2 x^2} \frac{\partial}{\partial x} \left( x^2 \kappa \frac{\partial \tilde{g}}{\partial x} \right)
\]

\( x = r / a : \text{co-expanding coordinate}, \quad a = \text{expansion factor}, \quad y = \ln p \)

**CRASH code in 1D spherical geometry:** Kang et al. 2013
The expansion factor, $a(t)$, is defined so that the shock position, $x_s = \text{constant}$, in co-expanding coordinate, while $r_s(t) = a(t) \cdot x_s$ expands in physical coordinate.

The shock position is fixed in the co-expanding frame.
Type Ia SNR Model: 1D spherical CRASH

\[ M_{ej} = 1.4 M_\odot, \quad E_o = 10^{51} \text{ ergs}, \quad n_{ISM} = 0.3 \text{ cm}^{-3}, \quad T_0 = 3 \times 10^4 \text{ K}, \quad B_0 = 5 \mu G \]

Two different models for \( B(x) \) and AD

\[
G(p) = \int_{r_i}^{r_f} 4\pi \cdot f(p) p^4 \cdot r^2 \, dr
\]
Highest energy end of CR spectra determines keV, GeV-TeV emission.

\[ E^2 N(E) : \text{volume integrated CR spectrum depends on } B(r) \text{ model.} \]

\[ B(r,t) \Rightarrow V_A(r,t) \& \kappa(p,r,t) \Rightarrow q : \text{slope} \]
\[ \& E_{\text{max},p}, E_{\text{max},e} (\text{cooling}) \]

So details of DSA modeling are important in predicting nonthermal emission from SNRs.
Tycho’s SNR

Projected X-ray emission

$B_2 = 100-500 \mu G$

Magnetic Field Amplification at shocks

Nonthermal emission from Tycho’s SNRs

$N_p(E) \propto E^{-2.3}$

$B_2 \sim 200\mu G$

$\pi^0$ decay emission

Giordano et al 2011

2014–10–10
-Wave-particle interactions are important in DSA theory: So Various plasma instabilities, MFA, Alfvenic drift, pre-heating of protons/electrons should be studied further by plasma simulations.

→ provide phenomenological models for DSA simulations

-Detailed plasma physics of DSA are important in predicting nonthermal emission from SNRs.

→ Testing “SNR hypothesis for the origin of Galactic CRs”
Proton acceleration at shocks: 2D Hybrid simulation

At parallel shocks, stream of accelerated protons into upstream
→self-generated waves
→Turbulent B amplification

At perpendicular shocks
No accelerated protons into upstream
→No turbulent waves

**DSA** = Diffusive Shock Acceleration
at quasi-parallel shocks ($\theta_B < 45^\circ$)

**SDA** = Shock Drift Acceleration
at quasi-perpendicular shocks ($\theta_B > 45^\circ$)

At quasi-parallel shocks
- CR acceleration efficiency
  ~10-20% in energy
- CR injection efficiency
  ~$10^{-3}$ in number
Nonthermal radiation from CRs accelerated at SNR shocks ➔ provide observational evidence and constraints for CR acceleration.

- CR $e + B$ field ➔ Synchrotron (radio – X-ray)
- thermal & non-thermal bremsstrahlung
- CR $e +$ CMBR ➔ Inverse Compton scattering ➔ TeV $\gamma$-ray
- CR $p + p$ ➔ $\pi^0$ decay ➔ 100 GeV $\gamma$-ray

GeV-TeV $\gamma$-ray: Hadronic vs. Leptonic origin (protons vs. electrons)

In addition, emission lines in thermal X-rays

From Ellison’s talk
Monte Carlo Simulations with a scattering model:
- scattered with a prescribed scattering model,
- assume a steady-state shock structure with FEB
  e.g. Ellison, Baring, Jones, Vladimirov +

*Semi-analytic approach
- analytical solution of the stationary diffusion–convection equation
  + gasdynamic conservation equations
  e.g. Blasi, Amato, Caprioli, Molino +

*Time-dependent diffusion-convection Simulations
- diffusion approximation based on isotropy of particle distribution
- follow time dependent evolution of $f(x,p,t)$ + gasdynamics Eqs
  e.g. Berezhko et al., Kang & Jones

*Full PIC & Hybrid plasma simulations: 3D is required, non-relativistic
- follow individual particles and magnetic fields
- provide the most complete pictures, but very expensive

hybrid (Giacalone 2013; Gargate & Spitkovsky 2011, Caprioli & Spitkovsky 2014)
Energy Spectrum of Cosmic Rays are power-law of $\sim E^{-3}$

Below $10^{17.5}$ eV: Galactic CRs are nuclei accelerated in SNRs if magnetic fields are amplified to $B_1 \sim 30 \, \mu G$ at the shock.

Magnetic field: $B_1 \sim 30 \, \mu G$

At SNRs $E_{\text{max}} \approx Z \cdot 10^{15.5} \, \text{eV} \left( \frac{B_1}{30 \, \mu G} \right)$

Below $10^{17.5}$ eV: Galactic CRs are nuclei accelerated in SNRs if magnetic fields are amplified to $B_1 \sim 30 \, \mu G$ at the shock.