Effects of magnetic field and radiation transfer for the formation and evolution of circumstellar disks

Yusuke Tsukamoto

Nagoya-U

JSPS PD


Surface density

Temperature
Outline

- Smoothed Particle Hydrodynamics (SPH)
  - Why SPH?
  - Magnetic field and radiation transfer for SPH

- Numerical results
  - Protostar and disk formation with non-ideal radiation magneto-hydrodynamics simulations
  - Disk evolution with radiation hydrodynamics simulation

- Summary
Outline

- Smoothed Particle Hydrodynamics (SPH)
  - Why SPH?
  - Magnetic field and radiation transfer for SPH

- Numerical results
  - Protostar and disk formation with non-ideal radiation magneto-hydrodynamics simulations
  - Disk evolution with radiation hydrodynamics simulation

- Summary
Why smoothed particle hydrodynamics is used?

- We must follow large dynamic range (e.g., structure formation, galaxy formation, star and disk formation)

Adaptive method is required
- Adaptive mesh refinement (AMR)
- Smoothed Particle Hydrodynamics (SPH)
What is SPH?

Monte Carlo sampling for integration

\[
\langle f \rangle (x) = \int W(x - x'; h) f(x') dx'
\]

\[
\langle \nabla f \rangle (x) = \int \{ \nabla W(x - x') \} f(x') dx'
\]

\[
\frac{\rho}{D} + \rho \nabla \cdot \mathbf{v} = 0
\]

\[
\rho \frac{D\mathbf{v}}{Dt} = -\nabla p
\]

\[
\rho \frac{D}{Dt} \left( \frac{e}{\rho} \right) = -p \nabla \cdot \mathbf{v}
\]

\[
\rho(r) = \sum_{j=1}^{N_{\text{neigh}}} m_j W(|r - r_j|, h).
\]

\[
\frac{Dv_i}{Dt} = -\sum_{j=1}^{N} m_j \left( \frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla W(r_{ij}, h_i)
\]

\[
\frac{Du_i}{Dt} = \sum_{j=1}^{N} \left( \frac{p_i}{\rho_i^2} + \frac{1}{2} \Pi_{ij} \right) m_j v_{ij} \cdot \nabla W_{ij}.
\]
Pure hydrodynamics is not enough!

\[
\frac{D\rho}{Dt} = -\rho \partial^\mu v^\mu \\
\rho \frac{Dv^\mu}{Dt} = -\partial^\nu P \\
\rho \frac{De}{Dt} = -\partial^\nu P v^\nu
\]

Equation of continuity

Equation of motion

Energy equation of matter
Pure hydrodynamics is not enough!

- **Equation of continuity**
  \[
  \frac{D\rho}{Dt} = -\rho \partial^\mu v^\mu
  \]

- **Equation of motion**
  \[
  \frac{Dv^\mu}{Dt} = \partial^\nu T^{\mu\nu}
  \]

- **Energy equation of matter**
  \[
  \frac{De}{Dt} = \partial^\nu T^{\mu\nu} v^\nu
  \]

- **Induction equation**
  \[
  \frac{D}{Dt} \left( \frac{B^\mu}{\rho} \right) = \frac{B^\nu}{\rho} \partial^\nu v^\mu
  \]

\[
T^{\mu\nu} = -\left( p + \frac{B^2}{2} \right) \delta^{\mu\nu} + B^\mu B^\nu
\]

\[
e = \frac{1}{2} v^2 + u + \frac{B^2}{2\rho}
\]
Pure hydrodynamics is not enough!

\[
\frac{D\rho}{Dt} = -\rho \partial^\mu \nu^\mu \quad \text{Equation of continuity}
\]

\[
\rho \frac{Dv^\mu}{Dt} = \partial^\nu T^{\mu\nu} + \frac{\chi_F \rho}{c} F^\mu \quad \text{Equation of motion}
\]

\[
\rho \frac{De}{Dt} = \partial^\nu T^{\mu\nu} \nu^\nu - 4\pi \kappa_p \rho B + c \kappa_E \rho^2 \xi
\quad \text{Energy equation of matter}
\]

\[
\frac{D\xi}{Dt} = \partial^\nu F^\nu - \partial^\mu \nu^\nu P^{\mu\nu}_{\text{rad}} - 4\pi \kappa_p \rho B + c \kappa_E \rho^2 \xi
\quad \text{Equation of radiation energy}
\]

\[
\frac{D}{Dt} \left( \frac{B^\mu}{\rho} \right) = \frac{B^\nu}{\rho} \partial^\nu \nu^\mu \quad \text{Induction equation}
\]

\[
T^{\mu\nu} = -\left( p + \frac{B^2}{2} \right) \delta^{\mu\nu} + B^\mu B^\nu
\quad e = \frac{1}{2} \nu^2 - \frac{B^\mu B^\mu}{2}
\]
Pure hydrodynamics is not enough!

\[
\frac{D\rho}{Dt} = -\rho \partial^\mu v^\mu \quad \text{Equation of continuity}
\]

\[
\rho \frac{Dv^\mu}{Dt} = \partial^\nu T^{\mu\nu} + \frac{\chi_F \rho}{c} F^\mu \quad \text{Equation of motion}
\]

\[
\rho \frac{De}{Dt} = \partial^\nu T^{\mu\nu} v^\nu - 4\pi \kappa_p \rho B + c \kappa_E \rho^2 \xi
\]

\[
- \nabla \cdot \left[ \left\{ \eta_O (\nabla \times B) + \eta_H (\nabla \times B) \times \hat{B} - \eta_A (\nabla \times B) \times \hat{B} \times \hat{B} \right\} \times B \right]
\]

\[
\rho \frac{D\xi}{Dt} = \partial^\nu F^\nu - \partial^\mu v^\nu P^{\mu\nu}_{\text{rad}} - 4\pi \kappa_p \rho B + c \kappa_E \rho^2 \xi
\]

\[
\frac{D}{Dt} \left( \frac{B^\mu}{\rho} \right) = \frac{B^\nu}{\rho} \partial^\nu v^\mu \quad \text{Induction equation}
\]

\[
- \frac{1}{\rho} \nabla \times \left\{ \eta_O (\nabla \times B) + \eta_H (\nabla \times B) \times \hat{B} - \eta_A (\nabla \times B) \times \hat{B} \times \hat{B} \right\}
\]

\[
T^{\mu\nu} = - \left( p + \frac{B^2}{2} \right) \delta^{\mu\nu} + B^\mu B^\nu \quad e = \frac{1}{2} v^2 + u + \frac{B^2}{2\rho}
\]

**MHD**

**RHD**

**Non-ideal**
All physical processes have been successfully formulated for SPH!

- RHD (w FLD): Whitehouse+ (04,05,06)
- MHD: Price+ (04,05,11)
- Godunov type MHD: Iwasaki+ (12)
- Non-ideal MHD: Tsukamoto+ (13)
- Ideal RMHD: Bate+(14)

Today, we will show the world-first results of non-ideal RMHD simulations with SPH
Outline

- Smoothed Particle Hydrodynamics (SPH)
  - Why SPH?
  - Magnetic field and radiation transfer for SPH

- Numerical results
  - Protostar and disk formation with non-ideal radiation magneto-hydrodynamics simulations
  - Disk evolution with radiation hydrodynamics simulation

- Summary
Importance of magnetic field in star formation

- Outflow formation (Tomisaka 98, 02, Machida + 07)
- Magnetic Braking suppresses disk formation (Mellon + 07, 09, Li + 12)

「Magnetic Braking Catastrophe」

Mellon + 2007

Li + 2011
Problem of numerical simulations of previous works (Li+11,13 Mellon+07)

- **Inner boundary is relatively large (~6 AU)**
  - Small scale structure within a few inner boundary radius (~30 AU) would be affected by boundary
Problem of numerical simulations of previous works (Li+11, 13 Mellon+07)

- **Inner boundary is relatively large (~6 AU)**
  - Small scale structure within a few inner boundary radius (~30 AU) would be affected by boundary
  - They neglected formation and evolution of first cores
    - Magnetic diffusions are effective in first cores
Numerical Models

- **EOS**: $X=0.8, Y=0.28$, Ortho-Para ratio: 3:1 (Tomida+ 13)
- **Opacity model**: Semenov(03) + Ferguson (05)
- **Magnetic diffusion model**: Chemical network in gas phase + charged dust (a=0.035\(\mu\)m)

Cosmic ray ionization rate is \(\xi_{cr} = 10^{-17} \text{ s}^{-1}\)

- **Initial conditions**: equidensity molecular cloud core
- **models**: Ideal, Ohm, Ohm+Ambipolar

### Equations

\[
\alpha = \frac{E_{th}}{E_{grav}} = 0.3 \quad \beta = \frac{E_{rot}}{E_{grav}} = 0.01 \quad \frac{\Phi}{\Phi_{crit}} = 4.0
\]
Ideal density

Plasma beta

$\beta \sim 1$ at the center

$
\beta \sim 1000$

$\beta > \sim 1000$

Larger first core
Radial profiles

- Rotationally supported disk does form due to the suppression of magnetic braking
- The size of disk is very small (~0.5 AU) at the formation epoch.

Rotation velocity
- Ideal
- Ohm
- Ohm + Ambi

Large rotation velocity

Infall velocity
- Ideal
- Ohm
- Ohm + Ambi

Almost zero infall velocity

Protostar

Density
- Ideal
- Ohm
- Ohm + Ambi

Solid: x direction
Dashed: z direction

Rotationally supported disk forms!
How does disk evolve further?
Outline

Smoothed Particle Hydrodynamics (SPH)
- Why SPH?
- Magnetic field and radiation transfer for SPH

Numerical results
- Protostar and disk formation with non-ideal radiation magneto-hydrodynamics simulations
- Disk evolution with radiation hydrodynamics simulation

Summary
How does disk evolve after protostar formation?

- The protostar is surrounded by disk just after its formation.
- Disk mass (0.1 Msolar) >> Proto star mass (0.01 Msolar)

Gravitational Instability is effective in the early phase of disk evolution!

What is the structure of gravitationally unstable disk?

Inutsuka+12
Steady structure of gravitationally unstable disk

\[ \dot{M} = -2\pi r v_r \Sigma = \text{const} \left( \propto r^0 \right) \]

\[ \left| \frac{d \ln \Omega}{d \ln R} \right| \propto \frac{c_s^2}{\Omega} \Sigma = \frac{1}{2\pi} \dot{M} \]

\[ Q = \frac{\Omega c_s}{\pi G \Sigma} \propto 1 \left( \propto r^0 \right) \]

We assume the power laws

\[ \Sigma \propto r^{n\Sigma} \quad T \propto r^{nT} \quad \alpha \propto r^{n\alpha} \quad \Omega \propto r^{n\Omega} \]

With rotation profile \( n_\Omega \) and energy equation, we can determine the structure of gravitationally unstable disks.
Structure of Gravitationally unstable disk

\[ \dot{M} = -2\pi rv_r \Sigma = \text{const} \ (\propto r^0) \]

\[ \left| \frac{d \ln \Omega}{d \ln R} \right| \alpha \frac{c_s^2}{\Omega} \Sigma = \frac{1}{2\pi} \dot{M} \]

\[ Q = \frac{\Omega c_s}{\pi G \Sigma} \sim 1 \ (\propto r^0) \]

\[ \left| \frac{d \ln \Omega}{d \ln R} \right| \alpha \frac{c_s^2}{\Omega} \Sigma \Omega^2 = \frac{32\sigma T^4}{3\tau}, \quad \Omega \propto r^{-1.5} \]

\[ \kappa = \kappa_0 T^2 \]

Does such a extreme structure really form?

→ Let’s check it with 3D RHD simulations

\[ \Sigma \propto r^{-3}, \ T \propto r^{-3}, \ \alpha \propto r^{4.5} \]
Numerical method

- Radiative SPH (no magnetic field)
- Sink (radius is 1 AU) is dynamically introduced when the protostar forms
- Starting from equidensity molecular cloud core

\[
\alpha = \frac{E_{\text{th}}}{E_{\text{grav}}} = 0.6 \quad \beta = \frac{E_{\text{rot}}}{E_{\text{grav}}} = 0.007
\]

Further evolution!
Overview of disk evolution with RHD simulations

- Molecular cloud core
- Spiral arms form → quasi steady against GI
- We calculated $10^4$ years after protostar formation
- Surface density
- Temperature
- 400AU
- ~ 10000AU
Radial profile (1)

Angular velocity

\[ \Omega \propto r^{-1.1} \]

\[ \Omega \propto r^{-1.5} \]

Q value

\[ Q \approx 1 \]
→ Quasi steady against GI

Disk is optically thick

Optically depth: \( \tau_z \)
Radial profile (2)

Temperature

- Solid lines: Power laws with $T \propto r^{-1.1}$
- Dashed lines: $T \propto r^{-2.2}$

Surface density

- Solid lines: Power laws with $\Sigma \propto r^{-1.65}$
- Dashed lines: $\Sigma \propto r^{-2.2}$

$\alpha$ value

- Solid lines: $\alpha \propto r^{3.3}$
- Dashed lines: $\alpha \propto r^{1.65}$

Equation:

$$\left| \frac{d \ln \Omega}{d \ln R} \right|^2 \propto \frac{c_s^2}{\Omega} \Sigma \Omega^2 = \frac{32 \sigma T^4}{3 \tau}$$
Why $T \propto r^{-1.1}$?
By detailed analysis of the numerical simulation, we found that $F_r \gg F_z, F_\phi$ and local heating is negligible.

This power law describes the disk structure formed in the simulation very well → radiation process is not local!
Outline

- Smoothed Particle Hydrodynamics (SPH)
  - Why SPH?
  - Magnetic field and radiation transfer for SPH
- Numerical results
  - Protostar and disk formation with non-ideal radiation magneto-hydrodynamics simulations
  - Disk evolution simulation with radiation hydrodynamics simulation
- Summary
Summary

- We have performed non-ideal RSPMHD simulations of protostar formation
  - We found that the magnetic braking catastrophe can be overcome with
    1. magnetic diffusions
    2. sufficient numerical resolution

- We have performed RSPH simulations of disk evolution (~10^4 years after protostar formation)
  - Local energy balance does not hold in the massive disk around low mass star.
  - Non-local radiative energy transfer determines the temperature structure.
Appendix
Is FLD approximation OK?

- Can FLD approx. capture the temperature structure well?
  → According to comparison test, error of sound velocity is factor 1.5 at maximum (Kuiper+13)
- We expect that it does not change the main conclusions significantly.

But confirmation with more sophisticated radiation transfer scheme (e.g., Hasegawa+12) would be important.
Artificial conductivity (Price+ 08)

- To treat the discontinuities (shock and contact discontinuity), numerical dissipations (viscosity and conductivity) are required for each discontinuity because it stem from microscopic physics.
- SSPH only includes numerical viscosity.
- If we introduce thermal conductivity, CD can be correctly captured.